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Abstract

摘要

The physics of de Sitter space is essential to our understanding of our cosmological past, present, and future. It forms the foundation for the statistical predictions of inflation in terms of quantum vacuum fluctuations that are being tested with cosmic surveys. In addition, the current expansion of the universe is dominated by an apparently constant vacuum energy, and we again find our universe described by a de Sitter epoch. Despite the success of our predictions for cosmological observables, conceptual questions of the nature of de Sitter abound and are exacerbated by technical challenges in quantum field theory and perturbative quantum gravity in curved backgrounds. In recent years, significant process has been made using effective field theory techniques to tame these breakdowns of perturbation theory. We will discuss how to understand the long-wavelength fluctuations produced by accelerating cosmological backgrounds and how to resolve both the UV and IR obstacles that arise. Divergences at

德西特空间物理学对我们理解宇宙的过去、现在与未来至关重要。它为暴胀关于量子真空涨落的统计预言奠定了基础，目前这些预言正接受巡天观测的检验。此外，当前宇宙膨胀由表观恒定的真空能量主导，我们的宇宙仍处于德西特阶段。尽管我们对宇宙学可观测对象的预言取得了成功，但关于德西特本质的概念性问题仍大量存在，弯曲背景下量子场论和微扰量子引力的技术难题进一步加剧了这些问题。近年来，利用有效场论技术解决微扰论的这些失效问题取得了重大进展。我们将讨论如何理解加速宇宙学背景产生的长波涨落，以及如何解决出现的紫外和红外障碍。

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e-mail: drgreen@physics.ucsd.edu long wavelengths are resummed by renormalization group (RG) flow in the EFT. For light scalar fields, the RG flow manifests itself as the stochastic inflation formalism. In single-field inflation, long-wavelength metric fluctuations are conserved outside the horizon to all-loop order, which can be understood easily in EFT terms from power counting and symmetries.

电子邮箱:drgreen@physics.ucsd.edu 长波长处的发散可通过有效场论中的重整化群 (RG) 流求和重聚。对于轻标量场，RG 流体现为随机暴胀形式。在单场暴胀中，长波度规涨落在视界外对所有圈阶都是守恒的，这一点可以很容易在有效场论框架下通过幂次计数和对称性得到解释。

Keywords

关键词

Effective field theory - de Sitter space - Inflation - Cosmology

有效场论 - 德西特空间 - 暴胀 - 宇宙学

Introduction

引言

The physics of de Sitter space forms an important pillar in our understanding of the universe. Structure in the universe is widely believed to have originated in the distance past during an approximately de Sitter phase, inflation, where the tools of quantum field theory (QFT) in curved space are essential for computing the statistical predictions of inflation. While tree-level calculations are in precise agreement with observational data, the theoretical foundations of these calculations are poorly developed compared to their analogues in flat space. These complications persist in our in attempts to understand the universe at late times. Observations of the expansion of the universe at low redshifts are consistent with the existence of a new phase de Sitter-like expansion. Our understanding of the universe in the current epoch is limited by the same technical complications as inflation and by a number of open questions about the origin and fate of the small nonzero vacuum energy.

德西特空间物理学是我们理解宇宙的重要支柱。目前普遍认为，宇宙结构起源于远古时期近似德西特的暴胀阶段，在此阶段中弯曲空间量子场论 (QFT) 工具对计算暴胀的统计预言至关重要。虽然树级计算与观测数据精确吻合，但相比平坦空间中的对应理论，这类计算的理论基础发展并不充分。这些复杂性也存在于我们理解晚期宇宙的尝试中。低红移处宇宙膨胀的观测结果与存在一个新的类德西特膨胀阶段一致。我们对当前宇宙纪元的理解，既受到与暴胀相同的技术问题限制，也受到关于微小非零真空能起源与命运的诸多开放性问题的限制。

In classical general relativity, de Sitter (dS) space presents few mysteries [89]. It is a maximally symmetric solution to Einstein’s equations, exhibiting a $SO(d,1)$ group of isometries in d spacetime dimensions. For understanding our own universe, we often focus on the patch of de Sitter described in terms of an expanding FRW solution,

在经典广义相对论中，德西特 (dS) 空间几乎不存在谜团 [89]。它是爱因斯坦方程的最大对称解，在 d 维时空下具有 $SO(d,1)$ 个等距群。为了理解我们自身所在的宇宙，我们通常关注以膨胀 FRW 解描述的德西特补丁，

$$ds^2 = g^{\mathrm{dS}}_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2 d\mathbf{x}^2 = a(\tau)^2\left(-d\tau^2 + d\mathbf{x}^2\right), \tag{1}$$

where $a(t) = e^{Ht}$ or $a(\tau)H = -1/\tau$.

其中 $a(t) = e^{Ht}$ 或 $a(\tau)H = -1/\tau$ 。

The quantum nature of de Sitter space is significantly more complicated, even for noninteracting quantum fields. The litany of conceptual and technical challenges begin with the ultraviolet (UV) limit, where the so-called trans-Planckian problem [22, 23, 91] suggests that even our choice of vacuum might be sensitive to Planck-scale physics. The questions persist as we move to long distances, where infrared (IR) divergences and secular growth challenge our notion of a perturbative expansion [1, 8, 24, 44, 49, 60, 67, 79, 83, 96, 97] . This problem is compounded at higher-loop order where our treatment of the UV and IR regimes manifests itself as additional divergences in loop diagrams. The predictions for the scalar metric fluctuations that sourced structure in the universe are sensitive to how we treat these regimes; therefore, our belief that inflation is consistent with observations is dependent on a satisfactory resolution to these challenges.

即使对于非相互作用量子场，德西特空间的量子性质也要复杂得多。一系列概念与技术挑战从紫外 (UV) 极限就开始出现: 所谓的跨普朗克问题 [22, 23, 91] 表明，我们对真空的选择甚至可能对普朗克尺度物理敏感。当我们转向长距离时，问题依然存在: 红外 (IR) 发散和久期增长对我们的微扰展开概念提出了挑战 [1, 8, 24, 44, 49, 60, 67, 79, 83, 96, 97] 。这个问题在高阶圈图中更加严重，我们对 UV 和 IR 区域的处理会在圈图中表现为额外的发散。造成宇宙原初结构的标量度规涨落的预言，依赖于我们对这些区域的处理方式；因此，暴胀与观测相容的结论，取决于这些挑战能否得到满意的解决。

Fortunately, effective field theory (EFT) [30, 47, 65, 77] provides a variety of tools to understand both short and long distance behaviors of de Sitter space. Fundamentally, the difficulty with cosmological backgrounds is that UV physics evolves to the IR through the expansion of the universe. This process does not violate the fundamental principles of EFT, like the decoupling of scales, but it does make their manifestation less transparent. Of course, we should expect these principles to hold in dS, given that our universe is currently in a dS-like phase, and we use EFT successfully to describe the world around us. Yet, it remains challenging to make sense of decoupling in contexts, like cosmological particle production, where the curvature of spacetime is essential to the physical process. Our goal in this chapter is to demonstrate that QFT and perturbative quantum gravity in dS can be recast in the familiar language of EFT where the resolutions to many of these problems have ready-made solutions.

幸运的是，有效场论 (EFT)[30, 47, 65, 77] 提供了多种工具来理解德西特空间的短程和长程行为。从根本上说，宇宙学背景的难点在于，随着宇宙膨胀，紫外物理会演化到红外区域。这个过程并不违反 EFT 的基本原理 (比如标度退耦)，但确实让这些原理的表现形式不再直观。鉴于我们的宇宙当前正处于类德西特阶段，且我们已经成功利用 EFT 描述周围的世界，我们当然应当期待这些原理在德西特空间中成立。然而，在宇宙学粒子产生这类时空曲率是物理过程核心的场景中，要理清退耦的逻辑依然困难。本章我们的目标是证明，德西特空间中的 QFT 和微扰量子引力可以被重写为我们熟悉的 EFT 语言，而许多这类问题都已有现成的解决方案。

The most basic insight that underlies the success of EFT is that the physics de Sitter is characterized by a single energy scale, H , the Hubble scale. First and foremost, the blue-shifting of modes as we evolve backward in time does not negatively impact perturbation theory (in the Bunch-Davies vacuum or a finite energy excitation thereof) because physical processes do not occur at these high energies. Instead, the energy scale associated with particle production is H , such that Planckian physics is exponentially suppressed when $H \ll M_{\rm pl}$, where $M_{\rm pl}$ is the reduced Planck mass. This observation is well-known, especially in inflationary phenomenology, where the amplitude of primordial non-Gaussianity is usually determined by power counting in H [29]. The implications are less obvious in loop diagrams, where one encounters logarithmic divergences that need to be regulated. However, if one defines the strength of couplings at the Hubble scale, the familiar RG from flat space is unnecessary, and all such logarithmic terms vanish. Many of these observations may even seem self-evident but can become obscured without effective regulators in dS [84].

EFT 取得成功背后最基本的洞见是: 德西特物理由单一能标，即哈勃能标 H 刻画。最重要的一点是，当我们反向时间演化时，模式的蓝移并不会对微扰论 (在邦奇-戴维斯真空或其有限能量激发下) 造成负面影响，因为物理过程不会在这些极高能标下发生。相反，粒子产生对应的能标是 H ，因此当 $H \ll M_{\rm pl}$ 时普朗克物理被指数压低，其中 $M_{\rm pl}$ 是约化普朗克质量。这一结论广为人知，尤其是在暴胀唯象学中，原初非高斯性的幅度通常由 H 的幂次计数确定 [29]。这一点的推论在圈图中并不那么直观，圈图中会出现需要正则化的对数发散。但如果我们在哈勃能标处定义耦合强度，就不需要平坦空间中我们熟悉的重整化群，且所有这类对数项都会消失。许多这类结论看起来甚至不证自明，但如果不在德西特空间中使用合适的正则化，就很容易变得模糊不清 [84]。

The second, and less obvious, outcome of the EFT approach to de Sitter [31] is that the superhorizon evolution of fields and composite operators can be organized by explicit power counting in powers of $k/(aH) \ll 1$. In the process, the degrees of freedom are redefined according to their scaling dimension in k , and time evolution is recast in the language of dynamical RG flow, so that the powers of $k/(aH)$ that appear follow from dimensional analysis. The EFT does not contain any relevant operators (in the RG sense), reproducing the long known result that corrections grow at most logarithmically [104, 105]. Logarithmic terms can be understood as operator mixing, while irrelevant terms decouple at late times. For massless scalars, an infinite number of operators can mix, giving rise to the framework of stochastic inflation as the master equation for this RG flow. For metric fluctuations, the all-order conservation of the adiabatic and tensor metric fluctuations follows from power counting, as the dimensions of these operators are fixed by symmetries and cannot be modified by RG.

德西特 EFT 方法 [31] 的第二个、也更不直观的结论是, 超视界尺度上的场和复合算符演化可以通过对 $k/(aH) \ll 1$ 的显式幂计数来组织。在此过程中，自由度会根据它们在 k 中的标度维度重新定义，时间演化被改写为动力学重整化群流的形式，因此出现的 $k/(aH)$ 幂次可由量纲分析得到。该 EFT 不包含任何 (RG 意义上的) 相关算符，印证了“修正最多仅对数增长”这一早就得到的结论 [104, 105]。对数项可理解为算符混合，而非相关项在晚期退耦。对于无质量标量场，无穷多算符可以发生混合，随机暴胀框架正是描述该 RG 流的主方程。对于度规涨落，绝热和张量度规涨落的全阶守恒可由幂计数导出，因为这些算符的维度由对称性固定，无法被 RG 改变。

The results in this chapter will be presented from the point of view of EFT, particularly soft de Sitter effective theory (SdSET) [31], applied to (in-in) cosmological correlators of scalar fields in fixed dS and metric fluctuations in single-field inflation. Many of the key results have been or can be derived from different perspectives, including the conventional perturbation theory [16, 17], the wavefunction of the universe [50], and/or the physics of the static patch [69, 70]. SdSET has the unique advantage that many nontrivial results when explained in terms of diagrams of the original theory become simple observations about dimensional analysis within the EFT. In addition, hard-to-interpret IR divergences in the full theory are traded for UV divergences in the EFT where they have a standard interpretation in terms of RG. Our emphasis on SdSET is similar to the role of the exact RG and EFT in Polchinski’s proof of renormalizability of $\lambda\phi^4$ in flat space [76]; although one can reach the main result by diagrammatic arguments [102], Polchinski’s exact RG makes the conceptual meaning of the result transparent and generalizes it to other theories. Our point of view in this chapter, as with many presentations of EFT, is that we will only claim to have fully understood a phenomena when it can be explained by symmetries and power counting.

本章的结果将从 EFT，尤其是软德西特有效理论 (SdSET)[31] 的角度呈现，应用于固定 dS 背景中标量场的 (in-in) 宇宙学关联函数，以及单场暴胀中的度规涨落。许多核心结论已经或可以从不同角度推导得到，包括传统微扰论 [16,17]、宇宙波函数 [50] 和/或静态补丁物理 [69,70]。SdSET 的独特优势在于，原本用原理论的图解释时十分非平凡的结论，在该 EFT 中只是简单的量纲分析结论。此外，full 理论中难以解释的 IR 发散，在 EFT 中被转换为 UV 发散，而后者在 RG 框架下有标准的物理解释。我们对 SdSET 的强调，类似于 Polchinski 证明平坦空间中 $\lambda\phi^4$ 可重整化时，精确 RG 和 EFT 所扮演的角色；尽管也可以通过图解论证得到核心结论 [102]，但 Polchinski 的精确 RG 让结论的概念意义清晰透明，还能推广到其他理论。和许多 EFT 的阐述一样，本章我们的观点是: 只有当一个现象能通过对称性和幂计数解释时，我们才能宣称完全理解了它。

This chapter will be organized as follows: In section ”Effective Theory in de Sitter,” we will discuss dS as an EFT where the relevant energy scale is the expansion rate $H \ll M_{\rm pl}$. Perturbation theory will be controlled by the small size of the expansion rate in a predictable way. We will specifically show how there is no trans-Planckian problem [22, 23, 91] unless we give up the idea that short-distance physics of de Sitter is similar to flat space (which would also contradict everyday experience). In section ”Soft de Sitter Effective Theory,” we discuss how to understand inflationary and dS backgrounds on scales much larger than the size of the cosmological horizon, H^{-1} . These are the scales that give rise to IR divergences and secular growth in traditional approaches to perturbation theory. We will introduce the SdSET to handle this regime, and we will see that the IR divergences of the full theory are replaced with EFT UV divergences, so that they can be resummed via renormalization group (RG) flow, following the usual EFT playbook. In section ”Light Scalars,” we apply these results to massless scalars and show how stochastic inflation arises from operator mixing in SdSET. In section ”Dynamical Gravity,” we then explain how the all-order conservation of the metric follows from power counting and discuss some implications for slow-roll eternal inflation. We conclude in section ”Conclusions.”

本章结构安排如下: 在“德西特有效理论”一节, 我们将 dS 讨论为一个相关能标是膨胀速率 $H \ll M_{\rm pl}$ 的 EFT。微扰论受膨胀速率的小量控制，行为可预测。我们将具体说明为何不存在跨普朗克问题 [22, 23, 91]，除非我们放弃“德西特的短程物理和平坦空间类似”这一观点 (而该观点也和日常经验相符)。在“软德西特有效理论”一节，我们讨论如何理解远大于宇宙学视界尺度 H^{-1} 的暴胀和 dS 背景。这些尺度正是传统微扰论方法中产生 IR 发散和长期增长的尺度。我们将引入 SdSET 来处理这个区间，我们会看到 full 理论的 IR 发散被替换为 EFT 的 UV 发散，因此可以遵循常规 EFT 的方案，通过重整化群 (RG) 流对它们进行重求和。在“轻标量”一节，我们将这些结果应用到无质量标量场，说明随机暴胀如何从 SdSET 的算符混合中产生。在“动力学引力”一节，我们解释度规的全阶守恒如何从幂计数得到，并讨论对慢滚永恒暴胀的一些启示。我们在“结论”一节总结本章内容。

Effective Theory in de Sitter

德西特空间中的有效理论

All discussions of de Sitter space start from the central premise that the curvature of dS is small in Planck units or $H \ll M_{\rm pl}$. This ensures that our classical solution for the background is under control and can be described geometrically, up to small perturbations. Without such an assumption, there is no controlled background geometry in which to discuss quantum fields or metric fluctuations. However, even with this assumption, it is still not necessarily obvious that quantum gravitational effects are always suppressed by at least $(H/M_{\mathrm{pl}})^2$.

所有关于德西特空间的讨论都从一个核心前提出发: 德西特空间的曲率在普朗克单位下很小，即 $H \ll M_{\mathrm{pl}}$ 。这保证了我们得到的背景经典解是可控的，在小扰动范围内可以用几何方法描述。没有这个假设，我们就无法得到可控的背景几何，也就没法讨论量子场或度规涨落。但即便满足这个假设，量子引力效应也未必一定至少被 $(H/M_{\mathrm{pl}})^2$ 压低，这一点并不显然。

The most unambiguous way to define the energies relevant to a given process is to calculate an observable quantity sensitive to the physical energy scale. To simplify the discussion, we will mostly consider the case of a scalar field ϕ of mass m and action (in the FRW slicing),

定义特定过程相关能量最明确的方法，是计算一个对物理能标敏感的可观测量。为简化讨论，我们大多会考虑标量场 ϕ 的情况，其质量为 m ，作用量 (在 FRW 切片下) 为

$$S = \int dt d^3x a^3(t) \left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{a(t)^2} \partial_i \phi \partial^i \phi - m^2 \phi^2 \right], \tag{2}$$

where the spatial indices are raised with the Kronecker δ^{ij} . Following standard canonical quantization, we decompose the field in terms of Fourier modes, $\mathbf{k}$, according to

其中空间指标用克罗内克 δ^{ij} 升阶。按照标准正则量子化方法，我们将场按傅里叶模 $\mathbf{k}$ 分解，分解式为

$$\phi(\mathbf{x}, \tau) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\overline{\phi}^*(\mathbf{k}, \tau) a^\dagger_{\mathbf{k}} + \overline{\phi}(\mathbf{k}, \tau) a_{-\mathbf{k}} \right), \tag{3}$$

where

其中

$$\overline{\phi}(\mathbf{k}, \tau) = -ie^{i\left(v+\frac{1}{2}\right)\frac{\pi}{2}} \frac{\sqrt{\pi}}{2} H(-\tau)^{3/2} H^{(1)}_v(-k\tau), \tag{4}$$

is a solution to the classical equations of motion, where $v = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$ and $H^{(1)}_v$ is the Hankel function of the first kind. Next, we promote $a^\dagger_{\mathbf{k}}$ and $a_{\mathbf{k}}$ to quantummechanical operators that act on the vacuum state. The choice of vacuum is often presented as an ambiguity unique to de Sitter; however, to be consistent with physical expectations and experience in our own universe, we will require that when wavelength of the modes is subhorizon, $k \gg aH$, we reproduce the vacuum of flat space. Specifically, this means that as $\tau \to -\infty, \phi$ should behave as a field operator in flat space, namely, that $a^\dagger_{\mathbf{k}}$ creates a particle from the vacuum while $a_{\mathbf{k}}$ annihilates the vacuum. We can see that Equation (4) is a negative frequency mode by expanding the Hankel function in $\tau \to -\infty$,

是经典运动方程的一个解，其中 $v=\sqrt{\frac{9}{4}-\frac{m^2}{H^2}}$ 和 $H_v^{(1)}$ 是第一类汉克尔函数。接下来，我们将 $a_{\mathbf{k}}^{\dagger}$ 和 $a_{\mathbf{k}}$ 提升为作用于真空态的量子力学算符。真空的选择通常被认为是德西特空间独有的不确定性；然而，为了与我们对自身宇宙的物理预期和经验保持一致，我们要求当模式的波长处于视界内时，即 $k \gg aH$，我们可以得到平直空间的真空。具体而言，这意味着在 $\tau \rightarrow -\infty, \phi$ 条件下，它应当表现得如同平直空间中的场算符，即 $a_{\mathbf{k}}^{\dagger}$ 从真空中产生粒子，而 $a_{\mathbf{k}}$ 湮灭真空。通过在 $\tau \rightarrow -\infty$ 下展开汉克尔函数，我们可以看出方程 (4) 是一个负频率模式，

$$\bar{\phi} \rightarrow -i\frac{H(-\tau)}{\sqrt{2k}}\exp(-ik\tau). \tag{5}$$

This takes the form of a WKB solution for negative frequency mode if we identify the physical (WKB) frequency as

如果我们将物理 (WKB) 频率取为如下形式，它就符合负频率模的 WKB 解形式

$$\omega_{\text{physical}}(t)=\frac{k}{a(t)} \rightarrow \int^t dt'\omega(t')=\int^t dt'\frac{k}{a(t')}=-\frac{k}{a(t)H}=k\tau. \tag{6}$$

In this sense, using the canonical commutation relation for $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^{\dagger}$,

在这个意义上，对 $a_{\mathbf{k}}$ 和 $a_{\mathbf{k}}^{\dagger}$ 使用正则对易关系，

$$\left[a_{\mathbf{k}}^{\dagger}, a_{\mathbf{k}'}\right]=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}') \quad a_{\mathbf{k}}|0\rangle=\langle 0 \mid a_{\mathbf{k}}^{\dagger}=0 \quad , \tag{7}$$

reproduced flat space physics on short distances where $k \gg aH$. We may choose other states, corresponding to excitations of the flat space vacuum. For most such choices, the energy density also breaks the de Sitter symmetry and redshifts away through the expansion of the universe. The exceptions are the α -vacua [4,71], which are de Sitter invariant but correspond to infinite energy configurations from the flat space perspective and may be ill-defined when including interactions [12]. Arguably, the most important nontrivial excited state for the purpose of cosmology is a time-dependent background for the scalar field, $\phi(\mathbf{x},t)=\phi_0(t)$. A case of particular interest is inflation [13], where the energy density of the time evolution is well above the Hubble scale, $\dot{\phi}_0^2 \gg H^4$.

就能在 $k \gg aH$ 满足的短距离处重现平坦空间物理。我们可以选择其他态，对应平坦空间真空的激发态。对大多数这类选择来说，能量密度也会破坏德西特对称性，并随着宇宙膨胀红移消失。例外是 α 真空 [4,71]，这类真空满足德西特不变性，但从平坦空间的视角来看对应无穷能构型，在包含相互作用时可能是不适定的 [12]。可以说，对宇宙学研究而言，最重要的非平庸激发态是标量场的含时背景 $\phi(\mathbf{x},t)=\phi_0(t)$。一个特别受关注的例子是暴胀 [13]，其中含时演化的能量密度远高于哈勃能标 $\dot{\phi}_0^2 \gg H^4$。

For cosmological applications (see, for example, [13]), we are particularly interested in the case of massless scalars, $m^2=0\,(v=3/2)$ where

对于宇宙学应用 (例如参见 [13])，我们尤其关注无质量标量的情况，即 $m^2=0\,(v=3/2)$ 满足

$$\overline{\phi}(\mathbf{k},\tau) \to \frac{H}{\sqrt{2k^3}}(1-ik\tau)\,e^{ik\tau}, \tag{8}$$

and the equal-time two-point function for superhorizon modes ($k\tau \ll 1$) becomes

此时超视界模 ($k\tau \ll 1$) 的等时两点关联函数变为

$$\langle \phi(\mathbf{k},\tau)\,\phi(\mathbf{k}',\tau)\rangle = \frac{H^2}{2k^3}(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\,. \tag{9}$$

Although this is a quantum-mechanical calculation, the commutator $\left[\dot{\phi},\phi\right] \propto a^{-3}(t)$ vanishes at long wavelengths, and $\phi(-k\tau \ll 1)$ becomes an effectively classical statistical fluctuation. Essentially the same behavior is found for the tensor fluctuations of the metric, $g_{\mu\nu} = g^{\mathrm{dS}}_{\mu\nu} + \gamma_{\mu\nu}$, and repeating this calculation gives their power spectrum

尽管这是一次量子力学计算，但对易子 $\left[\dot{\phi},\phi\right] \propto a^{-3}(t)$ 在长波长处消失，$\phi(-k\tau \ll 1)$ 成为有效的经典统计涨落。我们在度规的张量涨落 $g_{\mu\nu} = g^{\mathrm{dS}}_{\mu\nu} + \gamma_{\mu\nu}$ 中也发现了完全相同的行为，重复该计算即可得到它们的功率谱

$$\left\langle \gamma^{s}(\mathbf{k},\tau)\,\gamma^{s'}(\mathbf{k}',\tau)\right\rangle = \frac{2H^2}{M_{\mathrm{pl}}^2}\frac{1}{k^3}(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\,\delta_{s,s'}, \tag{10}$$

where s, s' are the two helicities of the graviton. The small amplitude of the metric fluctuations confirms our intuition that when $H \ll M_{\mathrm{pl}}$ the metric is well described by the classical geometry, $g^{\mathrm{dS}}_{\mu\nu}$, up to small perturbations.

其中 s, s' 是引力子的两种螺旋度。度规涨落的小振幅验证了我们的直觉: 当 $H \ll M_{\mathrm{pl}}$ 时，度规在小扰动范围内可以用经典几何 $g^{\mathrm{dS}}_{\mu\nu}$ 很好地描述。

Although these calculations may be consistent with our intuition, the assumptions about early times may not seem so innocuous, as often articulated in terms of the trans-Planckian problem [22, 23, 91] . The source of concern is that we are defining the vacuum of the field in the far past, $\tau \to -\infty$, where the energy of a mode diverges, $\omega = -k\tau \to \infty$. Clearly, for any $k > 0$, there exists a time in the past where $\omega > M_{\mathrm{pl}}$, and, naively, there is a breakdown in our EFT. While potentially concerning, this is not necessarily a problem for the following reason: the energy of a single particle is not a Lorentz invariant quantity. It is therefore not a given that when $\omega \gg M_{\mathrm{pl}}$ there must be a breakdown of EFT, as it depends on the coordinate system. We can always go to some boosted coordinate system where then energy is below the Planck scale. This is familiar from flat space, where EFT breaks down when a Lorentz invariant quantity, like the center of mass energy $s = (\omega_1 + \omega_2)^2 - (\mathbf{k}_1 + \mathbf{k}_2)^2$, is larger than the cutoff of the EFT (the Planck scale in this case).

尽管这些计算可能符合我们的直觉，但对早期宇宙的假设并非看起来那样无害，这一点通常以跨普朗克问题 [22, 23, 91] 的形式被提出。担忧的根源在于我们在极远过去 $\tau \to -\infty$ 定义了场的真空，而在那里模式的能量发散 $\omega = -k\tau \to \infty$ 。显然，对任意 $k > 0$ ，过去都存在某个时刻满足 $\omega > M_{\rm pl}$ ，因此从朴素角度来看，我们的有效场论 (EFT) 发生了破缺。尽管这一问题潜在值得关注，但它未必真的是个问题，原因如下: 单个粒子的能量并非洛伦兹不变量，因此不能直接断定当 $\omega \gg M_{\rm pl}$ 时 EFT 一定会破缺，这取决于坐标系。我们总能换到某个 boost 坐标系，其中粒子能量低于普朗克尺度。这在平直空间中是很常见的: 当质心系能量 $s = (\omega_1 + \omega_2)^2 - (\mathbf{k}_1 + \mathbf{k}_2)^2$ 这类洛伦兹不变量大于 EFT 的截断 (本案例中为普朗克尺度) 时，EFT 才会破缺。

The challenge in dS is that there is no obvious analogue of the Mandelstam variables s, t , and u that we can use to diagnose the breakdown of EFT. Instead, we can simply look at what scales appear in our cosmological observables and whether there is a well-defined expansion in a small parameter. In the cosmological setting, we will consider equal-time in-in correlation functions [104,105],

德西特空间 (dS) 中的难点在于，不存在我们可以用来诊断 EFT 破缺的、对应曼德尔斯坦变量 s, t 和 u 的明显类似量。相反，我们可以直接观测宇宙学观测量中出现的尺度，以及理论是否存在按小参数展开的良定义展开式。在宇宙学背景下，我们将考虑等时的 in-in 关联函数 [104,105],

$$\langle \text{in} | Q(t) | \text{in} \rangle =$$

$$\left\langle \bar{T} \exp\left[i \int_{-\infty(1+i\varepsilon)}^{t} H_{\text{int}}(t')\, dt' \right] Q_{\text{int}}(t)\, T \exp\left[-i \int_{-\infty(1-i\varepsilon)}^{t} H_{\text{int}}(t')\, dt' \right] \right\rangle, \tag{11}$$

where $H_{\text{int}}(t) = -\int d^3x a^3(t) \mathcal{L}_{\text{int}}(\phi_{\text{int}}(\mathbf{x}, t))$ is the interaction Hamiltonian, $\phi_{\text{int}}(\mathbf{x}, t)$ are the interaction pictures fields, and $Q(t)\,(Q_{\text{int}}(t))$ is some operator composed of $\phi(\mathbf{k}_i, t)\,(\phi_{\text{int}}(\mathbf{k}_i, t))$ with $k_i \ll aH(t)$ (An alternate approach to cosmological correlators is to first calculate wavefunction of the universe [55] and then subsequently determine the in-in correlators by the usual rules of quantum mechanics. This point of view has some advantages [19, 20] including making the connection to holography in AdS more transparent [54, 63, 64] and clarifying the origin of some IR divergences [6, 50] .). At first sight, the integrals run to $t = -\infty$ and would indicate that the super-Planckian modes contribute to this correlator. This is a red-hearing, as the integral is oscillatory and therefore these contributions can (and will) cancel out. To see this [18], we note that the integrals are defined by the $i\varepsilon$ prescription to ensure we are in the lowest energy state. As a result, the original contour (in time) is composed of two pieces, $\tau = \tau_0 + (1 \pm i\varepsilon)\tau$ where $\tau \in (\pm\infty, 0]$ and $\tau_0 < 0$. We can Wick rotate this contour, $(1 \pm i\varepsilon)\tau \to \pm i\tau_E$, to give a single anti-time-ordered correlator,

其中 $H_{\text{int}}(t) = -\int d^3x a^3(t) \mathcal{L}_{\text{int}}(\phi_{\text{int}}(\mathbf{x}, t))$ 是相互作用哈密顿量，$\phi_{\text{int}}(\mathbf{x}, t)$ 是相互作用绘景场，$Q(t)\,(Q_{\text{int}}(t))$ 是由 $\phi(\mathbf{k}_i, t)\,(\phi_{\text{int}}(\mathbf{k}_i, t))$ 与 $k_i \ll aH(t)$ 构成的某算符 (宇宙学关联量的另一种处理方法是先计算宇宙波函数 [55]，再通过量子力学的常规规则得到入-出关联量。该观点有若干优势 [19, 20] ，包括能更清晰地建立与反德西特空间全息术的联系 [54, 63, 64] ，并阐明部分红外发散的起源 [6, 50] 。)。乍看之下，积分范围延伸至 $t = -\infty$ ，说明超普朗克模式会对该关联量产生贡献。这是误导，因为积分具有振荡性，因此这类贡献能够 (也确实会) 相互抵消。要说明这一点 [18]，我们注意到积分由 $i\varepsilon$ 处方定义，以保证我们处于最低能态。因此，原始 (时间方向的) 围道由两部分构成，$\tau = \tau_0 + (1 \pm i\varepsilon)\tau$ 其中 $\tau \in (\pm\infty, 0]$ 和 $\tau_0 < 0$ 。我们可以对该围道做威克转动 $(1 \pm i\varepsilon)\tau \to \pm i\tau_E$ ，得到单个反时序关联量,

$$\langle \text{in} | Q(t) | \text{in} \rangle = \left\langle \bar{T} \left(Q_{\text{int}}(\tau_0) \exp\left[-\int_{-\infty}^{\infty} H_{\text{int}}(i\tau_E + \tau_0)\, a(i\tau_E + \tau_0)\, d\tau_E \right] \right) \right\rangle .$$

(12)

We can then calculate any correlator using the anti-time-ordered Green function, $\langle \bar{T}\phi(\mathbf{k}, \tau_E)\, \phi(\mathbf{k}', \tau_E') \rangle \propto e^{-k|\tau_E - \tau_E'|}$. Taking $\tau_E \to \pm\infty$ for any one mode will always produce an exponentially suppressed contribution to the correlator and thus does not suggest a breakdown in perturbation theory. Instead, the correlators are dominated by $k|\tau_E| = \mathcal{O}(1)$ or energies $\omega_{\text{physical}} = \mathcal{O}(H)$.

我们可以利用反时序格林函数 $\langle \bar{T}\phi(\mathbf{k}, \tau_E)\, \phi(\mathbf{k}', \tau_E') \rangle \propto e^{-k|\tau_E - \tau_E'|}$ 计算任意关联量。对任意模式取 $\tau_E \to \pm\infty$，其对关联量的贡献始终是指数压低的，因此不代表微扰论失效。相反，关联量由 $k|\tau_E| = \mathcal{O}(1)$ 或能量 $\omega_{\text{physical}} = \mathcal{O}(H)$ 主导。

The intuition that the relevant energy scale is H is also confirmed by explicit calculations. For example, a massless scalar with a derivative interaction,

相关能标为 H 的观点也得到了 explicit 计算的证实。例如，带导数相互作用的无质量标量场，

$$\mathcal{L}_{\text{int}} = \frac{1}{\Lambda^4} \left(\partial_\mu \phi \partial^\mu \phi \right)^2, \tag{13}$$

produces a four-point function

会产生四点函数

$$\langle \phi(\mathbf{k}_1)\, \phi(\mathbf{k}_2)\, \phi(\mathbf{k}_3)\, \phi(\mathbf{k}_4) \rangle = \frac{H^8}{\Lambda^4} \frac{P_6(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3 \mathbf{k}_4)}{16 (k_1 k_2 k_3 k_4)^3 k_t^5} (2\pi)^3 \delta\left(\sum \mathbf{k}_i \right), \tag{14}$$

where $k_t = (k_1 + k_2 + k_3 + k_4)$ and P_6 is a sixth-order polynomial given in [35]. It is noteworthy that the cutoff appears in the dimensionless ratio $(H/\Lambda)^4$ which controls the amplitude of the correlator. As a result, the correlators are approximately Gaussian when $H \ll \Lambda$ and consistent with weak coupling. The additional powers of H also arise from dimensional analysis as ϕ is a field of dimension one, while $k\tau$ is dimensionless. From these two observations, it is easy to see that H is the only energy scale beyond Λ needed to make the correct units in Equation (4). This situation becomes more complicated in inflationary backgrounds, where $\dot{\phi}^2 \gg H^4$, and we therefore require $\dot{\phi}^2 \ll \Lambda^4$ for control at weak coupling [34] (see Fig. 1 for illustration).

其中 $k_t = (k_1 + k_2 + k_3 + k_4)$ 和 P_6 是文献 [35] 中给出的六阶多项式。值得注意的是，截断出现在控制关联子振幅的无量纲比 $(H/\Lambda)^4$ 中。因此，当 $H \ll \Lambda$ 时，关联子近似为高斯分布，且与弱耦合一致。H 的额外幂次也来自量纲分析，因为 ϕ 是一维场，而 $k\tau$ 是无量纲的。由这两点可知，在方程 (4) 中，H 是除 Λ 外，唯一能得到正确单位所需的能量标度。这种情况在暴胀背景下会变得更复杂，此时满足 $\dot{\phi}^2 \gg H^4$，因此我们要求 $\dot{\phi}^2 \ll \Lambda^4$ 才能在弱耦合下保持可控 [34](示例见图 1)。

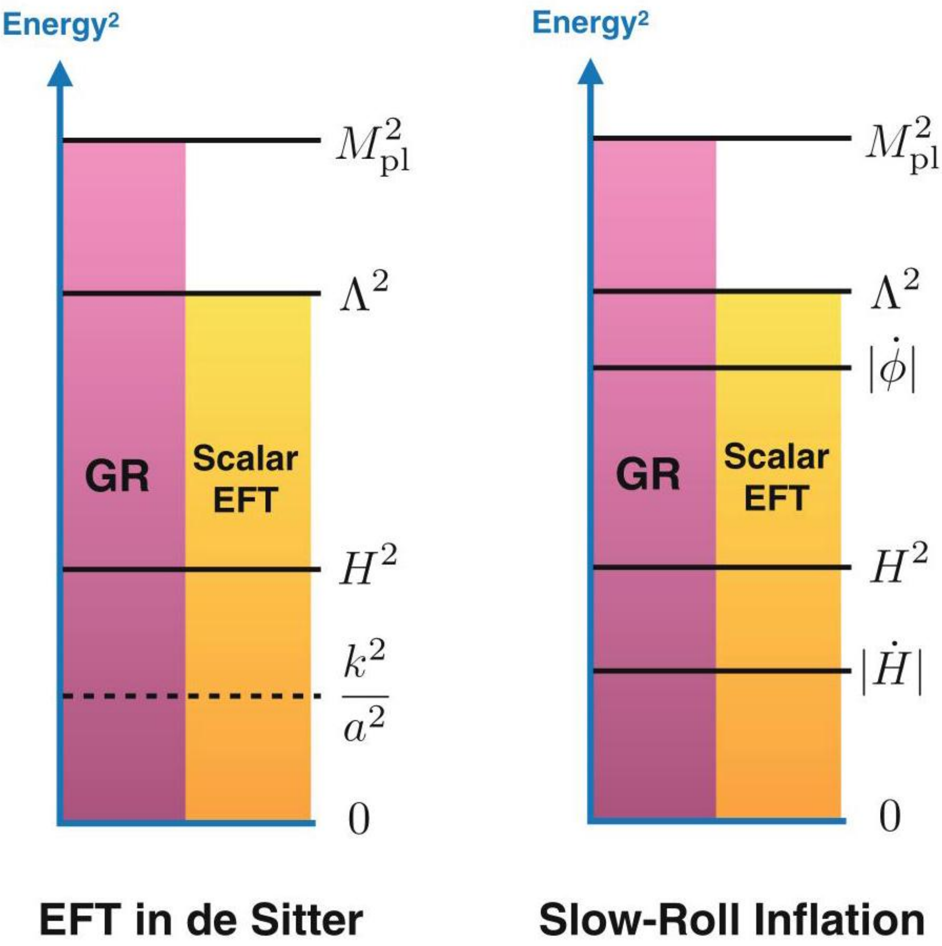

Fig. 1 The hierarchy of scales that defines EFT in de Sitter (Left) and slow-roll inflation (Right). For the background to be under control, we always require $H^2 \ll M_{\rm pl}^2$. In pure dS, scalar EFT is under control as long as $H^2 \ll \Lambda^2$. In contrast, in slow-roll inflation, the scalar background is controlled by $|\dot\phi| \gg H^2$ and therefore requires the stronger constraint that $|\dot\phi| \ll \Lambda^2$. This gives rise to the result that equilateral non-Gaussianity is small in slow roll, $f_{\rm NL}^{\rm eq} \ll 1$ [34]

图 1 定义了德西特空间 (左) 与慢滚暴胀 (右) 中有效场论的标度层级。为了让背景可控，我们始终要求 $H^2 \ll M_{\rm pl}^2$。在纯德西特空间中，只要满足 $H^2 \ll \Lambda^2$，标量有效场论就是可控的。与之相对，在慢滚暴胀中，标量背景由 $|\dot\phi| \gg H^2$ 控制，因此需要更强的约束条件 $|\dot\phi| \ll \Lambda^2$。由此可得结论: 慢滚暴胀中等边非高斯性很小，即 $f_{\rm NL}^{\rm eq} \ll 1$ [34]

Naturally, it seems reasonable to assume that the tools of EFT are applicable and under control in dS if the cutoff of the EFT is above the Hubble scale, $\Lambda \gg H$. Concretely, suppose we are given an EFT with an interaction Hamiltonian density

很自然地，我们有理由认为: 如果有效场论的截断高于哈勃标度 $\Lambda \gg H$，那么有效场论工具就适用于德西特空间且在该空间内可控。具体来说，假设我们有一个带相互作用哈密顿密度的有效场论

$$\mathcal{H}_{\rm int} = \sum_\Delta \frac{\lambda_\Delta}{\Lambda^{\Delta-4}} \mathcal{O}_\Delta \tag{15}$$

where we are working in $d = 4$ spacetime dimensions, and the sum is over the list of operators, $\mathcal{O}_\Delta$, organized by their (flat space) scaling dimension Δ. Then, at tree level, each term in the sum will contribute to a correlator whose dimensionless amplitude is $\lambda_\Delta (H/\Lambda)^{\Delta-4}$.

我们在 $d=4$ 维时空中进行研究，求和遍历按 (平坦空间) 标度维 Δ 整理的算符列表 $\mathcal{O}_\Delta$ 。此时在树图阶，求和中的每一项都会对关联子产生贡献，该关联子的无量纲振幅为 $\lambda_\Delta (H/\Lambda)^{\Delta-4}$ 。

The intuition that H sets the physical scale of de Sitter correlators is further confirmed at one loop. As with tree-level correlators, all the explicit units of energy are replaced with the appropriate powers of H, such that any large loop correlations arise from divergences in dimensionless integrals involving k and/or τ. Generic loop corrections will be discussed in section "Soft de Sitter Effective Theory," but we can gain significant intuition from specialized to the case $\upsilon = 1/2\left(m^2 = 2H^2\right)$, which is the conformal mass [52]. De Sitter space is conformally flat, and for this special value of the mass, certain field theories in de Sitter are simply the Weyl transform of a conformal field theory (CFT) involving a massless (The nonzero mass in dS arises from the curvature coupling needed to make the gravitational $T^\mu_\mu = 0$ for a free scalar in flat space.) scalar in flat space. Concretely, for an operator of dimension Δ, the relation between the dS and CFT correlators is given by

H 决定德西特关联子物理标度这一直觉在单圈水平得到了进一步验证。和树图关联子一样，所有明确的能量单位都被替换为 H 的适当幂次，因此任何大的圈关联都来自涉及 k 和/或 τ 的无量纲积分中的发散。我们会在“软德西特有效理论”一节讨论一般的圈修正，而通过专门研究共形质量的情况 $\upsilon = 1/2\left(m^2 = 2H^2\right)$ [52]，我们可以得到很多直观结论。德西尔空间是共形平坦的，对于这个特殊质量值，德西特空间中的某些场论就是共形场论 (CFT) 的外尔变换——该共形场论包含平坦空间中的无质量标量 (德西特空间中的非零质量来自曲率耦合，而该曲率耦合是得到平坦空间中自由标量的引力 $T^\mu_\mu = 0$ 所必需的)。具体而言，对于维数为 Δ 的算符，德西特关联子和共形场论关联子的关系由下式给出:

$$\langle \mathcal{O}_\Delta (\mathbf{x}, \tau) \ldots \rangle_{\mathrm{dS}} = (a(\tau))^{-\Delta} \langle \mathcal{O}_\Delta (\mathbf{x}, \tau) \ldots \rangle_{\mathrm{CFT}}. \tag{16}$$

One can calculate this explicitly in the case of $\lambda\phi^4$ in $d = 4 - \varepsilon$ dimensions, which is conformal for $\lambda = 0$ and $\lambda = \lambda_\star = \frac{16\pi^2}{3}\varepsilon$ (see, for example,[80]). The dimension of ϕ^2 in flat space is $\Delta = d - 2$ at $\lambda = 0$ and $\Delta = d - 2 + \varepsilon/3$ at the nontrivial fixed point $\lambda = \lambda_\star$.

我们可以在 $\lambda\phi^4$ 处于 $d = 4 - \varepsilon$ 维的情况下对此进行显式计算，该情况在 $\lambda = 0$ 和 $\lambda = \lambda_\star = \frac{16\pi^2}{3}\varepsilon$ 下是共形的 (例如参见文献 [80])。平坦空间中 ϕ^2 的维度，在 $\lambda = 0$ 时为 $\Delta = d - 2$，在非平凡不动点 $\lambda = \lambda_\star$ 处为 $\Delta = d - 2 + \varepsilon/3$ 。

Using the Weyl transformation from flat space, the de Sitter correlator at the nontrivial fixed point is necessarily

利用从平坦空间出发的外尔变换，非平凡不动点处的德西特关联函数必然为

$$\left\langle \phi^2 (\mathbf{k}, \tau)\, \phi^2 (\mathbf{k}', 0) \right\rangle_{\mathrm{dS}} \propto \frac{k^{1+\varepsilon/3}}{a(\tau)^{4-2\varepsilon/3}}, \tag{17}$$

where we have defined

此处我们定义了

$$\phi^n(\mathbf{k},\tau) = \int d^{3-\varepsilon}x e^{-i\mathbf{k}\cdot\mathbf{x}}\phi^n(\mathbf{x},\tau)$$

$$= \int \left(\prod_{i=1}^{n} \frac{d^{3-\varepsilon}k_i}{(2\pi)^{3-\varepsilon}}\phi(\mathbf{k}_i,\tau)\right)(2\pi)^{3-\varepsilon}\delta\left(\sum_{i=1}^{n}\mathbf{k}_i - \mathbf{k}\right). \tag{18}$$

Clearly, we must get the same answer by perturbation theory in dS. By direct calculation, the leading perturbative correction in λ gives

显然，德西特空间中的微扰论必定给出相同结果。通过直接计算，λ 中的领头阶微扰修正为

$$\langle\phi^2(\mathbf{k},\tau)\phi^2(-\mathbf{k},\tau)\rangle = \frac{c}{2a(\tau)^{2\Delta_{\phi^2}}}k^{1-\varepsilon} \tag{19}$$

$$\times\left[1 + \frac{\lambda}{32\pi^4 C}\left(\frac{1}{\varepsilon} + \log\left(\frac{\mu}{H}\right) - \log(-k\tau) + \ldots\right)\right].$$

where μ is the renormalization scale, $\Delta_{\phi^2} = d-2$, and $C \approx -1/\left(4\pi^2\right)$ is a constant. The key thing to notice is that the logarithmic divergence that can be separated into $\log\mu/H$ and $\log(-k\tau)$. Since the metric is unchanged by the rescaling $a \to \xi a$ and $x \to x/\xi$, it is required that powers of k and a must appear together as the physical wavenumber k/a [84]. Using $\tau = -1/(aH)$, one can see that any log should be rewritten in terms of these two dimensionless ratios. This is true more generally than this one example, of course, and can be seen a variety of loop corrections calculated in the literature [6, 24] . It follows that any $\log\mu/H$ can be set to zero by simply choosing $\mu = H$, which means that H is the correct physical scale we should associate with de Sitter correlators. With this choice, there is no conventional renormalization group flow; there are no large logs involving the renormalization scale.

其中 μ 是重整化标度，$\Delta_{\phi^2} = d-2$ ，$C \approx -1/\left(4\pi^2\right)$ 是常数。需要注意的关键是，对数发散可以被分离为 $\log\mu/H$ 和 $\log(-k\tau)$ 。由于度规在标度变换 $a \to \xi a$ 和 $x \to x/\xi$ 下保持不变，因此要求 k 和 a 的幂次必须共同组成物理波数 k/a [84]。利用 $\tau = -1/(aH)$ 可以发现，任何对数都可以改写为这两个无量纲比的形式。当然，这一结论比这个单独的例子更具一般性，在文献中计算的多种圈修正中都能得到验证 [6, 24] 。由此可得，任何 $\log\mu/H$ 都可以通过直接选取 $\mu = H$ 置零，这意味着 H 才是我们应当关联到德西特关联函数的正确物理标度。做出这一选择后，就不存在传统的重整化群流；也不会出现涉及重整化标度的大对数项。

One may be troubled that setting $\mu = H$ has left us with a large log, $\log(-k\tau)$. These are large infrared logs which we will handle by matching onto an EFT for the long modes, with $k/a \ll H$. Yet, for the case of $\lambda\phi^4$ in $d = 4-\varepsilon$, we know that this log must be resummed in order to match the result from the Weyl transformation in Equation (16). The procedure for doing this known as dynamical RG involves introducing an unphysical time, $\tau_\star$, to regulate the large logs. As with conventional RG, we define a renormalized operator $\mathcal{O}_\mathrm{R} = Z(k_\star\tau_\star)\mathcal{O}$ so that

有人可能会困扰，置零 $\mu = H$ 后留下了大对数项 $\log(-k\tau)$。这些是大红外对数，我们将通过匹配长模式的有效场论来处理，即通过 $k/a \ll H$。但对于 $\lambda\phi^4$ 在 $d = 4-\varepsilon$ 中的情况，我们知道必须对这个对数进行重求和，才能匹配式 (16) 中外尔变换得到的结果。这个被称为动力学重整化群的操作会引入非物理时间 $\tau_\star$，来正则化大对数。和传统重整化群一样，我们定义重整化算符 $\mathcal{O}_{\rm R} = Z(k_\star\tau_\star)\mathcal{O}$ 使得

$$\left\langle \phi_{\rm R}^2(\mathbf{k},\tau)\,\phi_{\rm R}^2(-\mathbf{k},\tau)\right\rangle = \frac{c}{2a(\tau)^{2\Delta_\phi}}k^{1-\varepsilon}\left[1+\frac{\lambda}{32\pi^4 C}\log\left(k_\star\tau_\star/(k\tau)\right)+\dots\right].$$

(20)

Since $\mathcal{O}$ is independent of $\tau_\star$, one can write a differential equation of

由于 $\mathcal{O}$ 与 $\tau_\star$ 无关，我们可以写出如下微分方程

$$\frac{d}{d\log(k_\star\tau_\star)}\left\langle \phi_{\rm R}^2(\mathbf{k},\tau)\,\phi_{\rm R}^2(-\mathbf{k},\tau)\right\rangle = \frac{\lambda}{8\pi^2}\left\langle \phi_{\rm R}^2(\mathbf{k},\tau)\,\phi_{\rm R}^2(-\mathbf{k},\tau)\right\rangle \tag{21}$$

Once we recognize that $k_\star\tau_\star/(k\tau)$ always appears together, we can write this as a differential equation in $k\tau$, whose solution is

一旦我们认识到 $k_\star\tau_\star/(k\tau)$ 总是共同出现，我们就可以将其写为关于 $k\tau$ 的微分方程，其解为

$$\left\langle \phi_{\rm R}^2(\mathbf{k},\tau)\,\phi_{\rm R}^2(-\mathbf{k},\tau)\right\rangle = \frac{1}{a(\tau)^{2\Delta_{\phi^2}+2\gamma_{\phi^2}}}k^{1-\varepsilon+2\gamma_{\phi^2}} \tag{22}$$

where $\gamma_{\phi^2} = \lambda_H/16\pi^2$, so that at the conformal fixed point $\lambda_\star = 16\pi^2\varepsilon/3$, we find exact agreement between Equations (22) and (16). Note that this procedure is nearly identical to the perturbative derivation of (16) in flat space, where one would use the Callan-Symanzik equations to convert a logarithmic correction (the anomalous dimension) into a power law.

其中 $\gamma_{\phi^2} = \lambda_H/16\pi^2$，因此在共形不动点 $\lambda_\star = 16\pi^2\varepsilon/3$ 处，我们发现式 (22) 和式 (16) 完全一致。注意这个过程和平坦空间中对 (16) 的微扰推导几乎相同，在平坦空间中人们会用卡伦-西曼齐克方程将对数修正 (反常维度) 转化为幂律。

These results suggest that we should think about resumming the large time-dependent logs in general [27, 46, 61, 83, 109] , and not just at the conformal fixed point. There is nothing about the calculation that suggests it is important that we are at the fixed point, particularly as the RG itself is unimportant after we set $\mu = H$. The result of this resummation is that we introduce an anomalous dimension for ϕ^2 , $\gamma_{\phi^2} = \lambda_H/16\pi^2$ where the coupling λ_H is the value of the coupling at $\mu = H$. This suggests a deeper connection between the large logs and anomalous dimensions, as we will see in the next section. These results are also closely related to results in the cosmological bootstrap program [15], particularly the non-perturbative structure of QFT correlators in dS [38,59]. We will not explore that connection here as our emphasis is EFT, particularly with an eye toward perturbative quantum gravity.

这些结果表明，我们应当考虑对一般 [27, 46, 61, 83, 109] 中依赖时间的大对数项重新求和，而非仅在共形不动点处进行。计算过程中没有迹象表明处于不动点是必要条件，尤其是在我们固定 $\mu = H$ 后，重整化群本身就已经不重要了。重新求和的结果是我们为 ϕ^2 引入了反常维数，即 $\gamma_{\phi^2} = \lambda_H/16\pi^2$ ，其中耦合 λ_H 是耦合在 $\mu = H$ 处的取值。这表明大对数项与反常维数之间存在更深层的联系，我们会在下一节展开讨论。这些结果也与宇宙自举程序的研究结果 [15] 密切相关，尤其是德西特空间中量子场论关联函数的非微扰结构 [38,59]。我们在此不对该联系做进一步探究，因为本文的研究重点是有效场论，具体来说是围绕微扰量子引力展开讨论。

The idea of dynamical RG as a solution to the IR behavior in dS has a long history [27, 39, 75] , and in many cases, it was known to be the most physically sensible solution to the problem of secular growth. Yet, we should note that the not every appearance of $\log(-k\tau)$ signals some universal correction to the long-distance physics; logs may also arise from finite scheme-dependent corrections to the parameters of the Lagrangian. For example, due to the challenge of finding good mass independent regulators, loop diagrams may introduce a finite shift of the mass of our field ϕ by $m^2 \to m^2 + \kappa H^2$ for some $\kappa = \mathcal{O}(\lambda)$. As m^2 changes the superhorizon scaling with $a(t)$, a perturbative correction to m^2 will take the form of logarithmic growth. However, unlike our usual expectations around logs from flat space, we can find schemes where $\kappa = 0$ so that these terms do not appear.

将动力学重整化群作为解决德西特空间红外行为的方案已有很长的研究历史 [27, 39, 75] ，并且在很多情况下，它都被认为是长期增长问题在物理上最合理的解决方案。但我们需要指出，并非只要出现 $\log(-k\tau)$ 就意味着长程物理存在某种普适修正；对数项也可能来源于对拉格朗日量参数的、依赖方案的有限修正。例如，由于很难找到良好的质量无关正则化方案，圈图可能会给我们的场质量 ϕ 带来一个大小为 $m^2 \to m^2 + \kappa H^2$ 的有限移动，其中 $\kappa = \mathcal{O}(\lambda)$ 为某个参数。当 m^2 改变随 $a(t)$ 变化的超视界标度行为时，对 m^2 的微扰修正会表现为对数增长。然而，和我们在平直空间中对对数项的一般预期不同，我们可以找到不存在这些项的方案，即 $\kappa = 0$ 。

Soft de Sitter Effective Theory

软德西特有效理论

From working with interacting scalars in a fixed de Sitter background, we have learned two essential features of EFT in de Sitter. The first is that cosmological correlators are a reflection of physics at the energy-scale H . This means that flat space power counting is a reasonably good guide for estimating the amplitude of cosmological correlators when we think of H as the only energy scale in the problem. The second, and less obvious lesson, is that there is still the potential for large corrections coming from lower energies/longer wavelengths, namely, where $k/a \ll H$ or $k\tau \ll 1$. Away from CFTs, our flat space intuition does not tell us how to anticipate results in this regime as it is where the curvature of spacetime is dominant.

通过研究固定德西特背景下的相互作用标量场，我们了解到德西特空间中有效理论 (EFT) 的两个核心特征。第一，宇宙关联算符反映了能量尺度 H 下的物理。这意味着当我们将 H 视为该问题唯一的能量尺度时，平坦空间幂次计数可以相当好地估算宇宙关联算符的振幅。第二，也是更不直观的结论: 更低能量/更长波长处仍然可能产生大修正，即对应 $k/a \ll H$ 或 $k\tau \ll 1$ 的情况。远离共形场论 (CFT) 时，我们的平坦空间直觉无法帮助我们预判该区域的结果，因为这正是时空曲率占主导的区域。

From an EFT point of view, we would like to understand any large long-distance effect as being a consequence of power counting in the usual RG sense. If the superhorizon theory is far from the behavior of a free theory, then we should be able to identify some relevant or marginal operators which are responsible. Concretely, our goal is to make the expansion and power counting in $k/(aH) \ll 1$ explicit where relevant, marginal, and irrelevant interactions are characterized by scaling with $(k/[aH])^n$ with $n < 0, n = 0$, and $n > 0$, respectively. To accomplish this goal, we will write an EFT, the soft de Sitter effective theory [31] (SdSET), where $[a(t)H] = \Lambda(t)$ is a time-dependent UV cutoff. We will integrate out modes with hard comoving momenta, $p > [aH]$. The physical scales relevant to SdSET are illustrated in Fig 2.

从有效理论的角度来看，我们希望将所有大尺度长程效应都理解为常规重整化群 (RG) 意义下幂次计数的结果。如果超视界理论和自由理论行为相去甚远，那么我们应当能够找到造成这一结果的相关或临界算符。具体而言，我们的目标是在 $k/(aH) \ll 1$ 下明确写出展开和幂次计数: 相关、临界、无关相互作用分别对应随 $(k/[aH])^n$ 、$n < 0, n = 0$ 、$n > 0$ 标度，以此完成分类。为了实现这一目标，我们构建了一个有效理论——软德西特有效理论 [31](SdSET)，其中 $[a(t)H] = \Lambda(t)$ 是随时间变化的紫外截断。我们会积分掉硬共动动量模式 $p > [aH]$ 。软德西特有效理论对应的相关物理标度如图 2 所示。

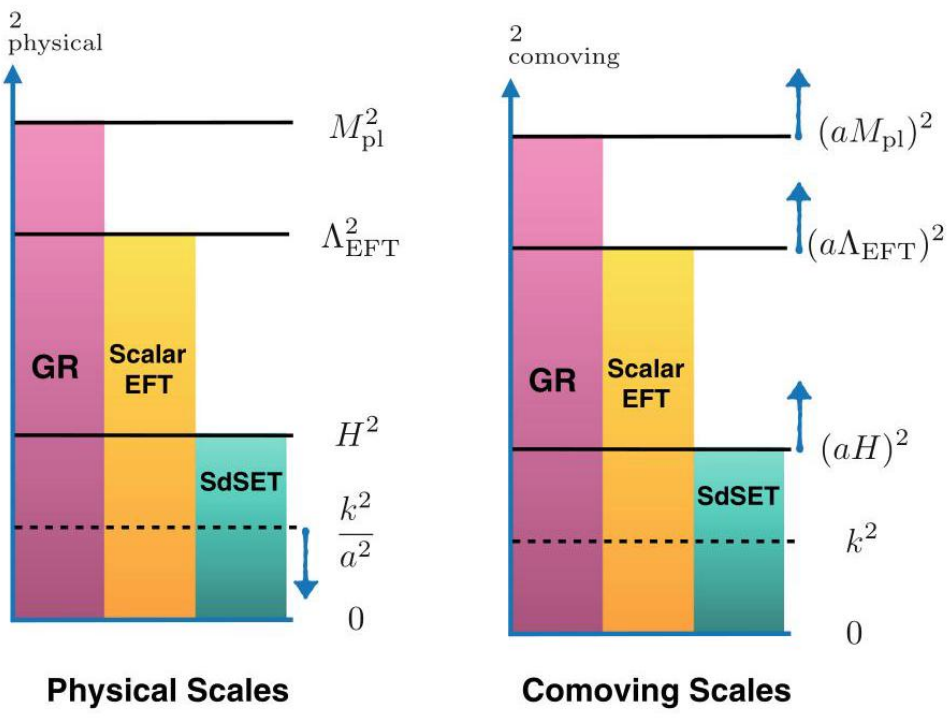

Fig. 2 The hierarchy of scales that defines the soft de Sitter effective theory in terms of physical angular frequencies (Left) and comoving angular frequencies (Right). The blue arrows indicate which scales evolve

under time evolution in each description

图 2 软德西特有效理论的标度层级，分别以物理角频率 (左) 和共动角频率 (右) 表示。蓝色箭头指示两种描述下哪些标度会随时间演化。

The central observation of SdSET that we will need is that ϕ does not behave like a single scaling operator in the long-wavelength limit, $k\tau \to 0$. If we simply set $k = 0$ and solve the equations of motion associated with the massive scalar in Equation (2), one finds two solutions

我们需要用到的 SdSET 核心结论是: 在长波长极限 $k\tau \to 0$ 下，ϕ 并不表现为单一标度算符。如果我们直接设 $k = 0$ 并求解方程 (2) 中有质量标量场对应的运动方程，会得到两个解

$$\phi(\mathbf{k} = 0, t) = \kappa_\alpha a(t)^{-\alpha} + \kappa_\beta a(t)^{-\beta}, \tag{23}$$

where $\kappa_{\alpha,\beta}$ are constants, $\alpha = \frac{3}{2} - \upsilon$, and $\beta = \frac{3}{2} + \upsilon$ so that $\alpha + \beta = 3$. Our goal is to define the theory by the scaling with $k/[a(t)H]$, but we see that ϕ does not behave as a single scaling operator in the $k \to 0$ limit. Following the usual EFT playbook, we want to organize our results in terms of explicit powers of the UV cutoff, $\Lambda = a(t)H$, and operators that scale like powers of k . Starting from the original scalar, one may then guess that we want to split the operator ϕ into φ_+ and φ_- using the ansatz

其中 $\kappa_{\alpha,\beta}$ 是常数，$\alpha = \frac{3}{2} - \upsilon$ ，且 $\beta = \frac{3}{2} + \upsilon$ 满足 $\alpha + \beta = 3$ 。我们的目标是用对 $k/[a(t)H]$ 的标度定义理论，但我们可以看到在 $k \to 0$ 极限下，ϕ 并不表现为单一标度算符。遵循常规有效理论的研究框架，我们需要将结果按紫外截断 $\Lambda = a(t)H$ 的明确幂次、以及按 k 幂次标度的算符来整理。从原始标量场出发，不难猜测我们需要利用假设将算符 ϕ 拆分为 φ_+ 和 φ_-

$$\phi = H\left(\varphi_+(\mathbf{x}, t)[a(t)H]^{-\alpha} + \varphi_-(\mathbf{x}, t)[a(t)H]^{-\beta}\right). \tag{24}$$

We have factored out the $k = 0$ solutions to the equations of motion so that powers of $[aH]$ will appear explicitly in the action after this change of variables (as we will see). In addition, the overall power of H is needed to match the units of ϕ as defined by the short-distance theory. With this choice, φ_+ and φ_- have the same scaling and engineering dimensions so that dimensionally $[\varphi_+(\mathbf{x}, t)] = [x]^{-\alpha}$ and $[\varphi_-(\mathbf{x}, t)] = [x]^{-\beta}$. This decomposition matches our expectations from an EFT expansion, as it will obviously break down if we try to extend it beyond the cutoff, $k > aH$, which coincides with subhorizon physics.

我们已经从运动方程中分解出了 $k = 0$ 解，因此完成这一变量替换后，$[aH]$ 的幂次会显式出现在作用量中 (我们稍后会看到)。此外，整体的 H 幂次是为了匹配短距离理论所定义的 ϕ 量纲。通过这一选择，φ_+ 与 φ_- 具有相同的标度和工程量纲，因此量纲分析可得 $[\varphi_+(\mathbf{x}, t)] = [x]^{-\alpha}$ 和 $[\varphi_-(\mathbf{x}, t)] = [x]^{-\beta}$ 。这种分解符合我们对 EFT 展开的预期，因为如果我们尝试将其延拓到截断 $k > aH$ 之外，展开显然会失效，而该截断恰好对应亚视界物理。

Starting with a free scalar field, we may plug this ansatz into the action to determine the action for φ_+ and φ_- :

从自由标量场出发，我们可以将这个 ansatz 代入作用量，推导出 φ_+ 和 φ_- 的作用量:

$$S = \int dtd^3x \left(-\upsilon \dot{\varphi}_+ \varphi_- + \upsilon \varphi_+ \dot{\varphi}_- + \frac{1}{[aH]^2} \partial_i \varphi_+ \partial^i \varphi_- + \ldots \right), \tag{25}$$

where $\mathrm{t} \equiv Ht$. At order leading order in derivatives, the resulting equations of motion are simply $\dot{\varphi}_\pm = 0$, which reflects the fact that our ansatz factored out the solutions to the equations of motion with $k = 0$. Terms with more than one time derivative, such as $\dot{\varphi}_\pm^2$, are removed using the leading equations of motion in order to maintain the correct number of degrees of freedom in the EFT [106]. Spatial derivatives always come suppressed by powers of aH and vanish in the limit $aH \to \infty$.

其中 $\mathrm{t} \equiv Ht$ 。在导数 leading 阶，所得运动方程可简化为 $\dot{\varphi}_\pm = 0$ ，这体现了我们的 ansatz 已经分解出了满足 $k = 0$ 的运动方程解。含有多于一个时间导数的项 (例如 $\dot{\varphi}_\pm^2$) 可以通过 leading 阶运动方程消去，以保证 EFT 中自由度的数量正确 [106]。空间导数始终受 aH 幂次压低，在 $aH \to \infty$ 极限下趋于零。

Unlike many EFTs in flat space, the coefficients of the low-energy action alone does not specify all the information we need regarding the microscopic (UV) theory. The action alone tells us that φ_+ commutes with itself and that the canonical commutator is $[\varphi_+(\mathbf{x}), \varphi_-(\mathbf{x}')] = -i\delta(\mathbf{x} - \mathbf{x}') / (2\upsilon)$. To match the correlators of the UV theory, we must therefore specify initial conditions for φ_+ in the form of classical stochastic variables. Concretely, the two point function of φ_+ that follows from Equation (4) implies that

和许多平直空间中的 EFT 不同，低能作用量的系数本身并不包含我们需要的关于微观 (紫外) 理论的全部信息。仅从作用量可知 φ_+ 与自身对易，正则对易子为 $[\varphi_+(\mathbf{x}), \varphi_-(\mathbf{x}')] = -i\delta(\mathbf{x} - \mathbf{x}') / (2\upsilon)$ 。因此，为了匹配紫外理论的关联函数，我们必须以经典随机变量的形式为 φ_+ 指定初始条件。具体而言，由式 (4) 得到的 φ_+ 两点函数满足:

$$\langle \varphi_+(\mathbf{k}) \varphi_+(\mathbf{k}') \rangle = \frac{C_\alpha^2}{k^3} k^{2\alpha} (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}'), \tag{26}$$

where $C_\alpha = 2^{1-\alpha} \frac{\Gamma\left(\frac{3}{2} - \alpha\right)}{\sqrt{\pi}}$, which we defined such that $C_\alpha = 1$ at $\alpha = 0$. The overall power of k^{-3} follows from the Fourier transform, while the power of $k^{2\alpha}$ shows that $\varphi_+(\mathbf{x})$ behaves as a scaling operator of dimension α . In fact, under the $SO(4,1)$ isometries of de Sitter space, $\varphi_\pm$ transform as primary operators of dimension α and β , respectively:

其中 $C_\alpha = 2^{1-\alpha} \frac{\Gamma\left(\frac{3}{2} - \alpha\right)}{\sqrt{\pi}}$ ，我们的定义满足在 $\alpha = 0$ 处 $C_\alpha = 1$ 。整体的 k^{-3} 幂次由傅里叶变换得到，而 $k^{2\alpha}$ 的幂次表明 $\varphi_+(\mathbf{x})$ 是量纲为 α 的标度算符。事实上，在德西特空间的 $SO(4,1)$ 等距变换下，$\varphi_\pm$ 分别是量纲为 α 和 β 的原生算符:

$$\varphi_\pm(\mathbf{x}, \mathrm{t}) \to \left[1 - 2\Delta_\pm x_i b^i + \left(x^2 - [aH]^{-2} \right) b_i \partial^i - 2b_i x^i \mathbf{x} \cdot \partial + 2b_i x^i \partial_\mathrm{t} \right] \varphi_\pm(\mathbf{x}, \mathrm{t}),$$

(27)

where $\Delta_+ \equiv \alpha$ and $\Delta_- \equiv \beta$. The observation that the solutions to the equations of motion behave like primary operators is well-known in both dS and AdS and is central to the (A)dS/CFT holographic dictionary [107]. The new development in SdSET is that the bulk action is expressed directly in terms of these operators.

其中 $\Delta_+ \equiv \alpha$ 和 $\Delta_- \equiv \beta$。运动方程的解表现为原生算符这一结论在 dS 和 AdS 中都是已知结论，也是 (A)dS/CFT 全息字典的核心 [107]。SdSET 的新进展在于，体作用量可以直接用这些算符表示。

Now, suppose we add interactions to the UV theory in the form of a potential,

现在，假设我们以势的形式给紫外理论引入相互作用，

$$V(\phi) = \sum_{n>3} \frac{\lambda_n}{n!} \phi^n \tag{28}$$

If we simply apply our ansatz, Equation (24), then the potential expressed in these variables becomes

如果我们直接应用我们的拟设 (即式 (24))，用这些变量表示的势将变为

$$V(\phi) = H^4 \sum_{n>3} \sum_{m=0}^{n} [aH]^{-(n-m)\alpha - m\beta} \frac{c_{n-m,m}}{(n-m)!m!} \varphi_+^{n-m} \varphi_-^m (\mathbf{x}) \tag{29}$$

where

其中

$$c_{n-m,m} = \lambda_n H^{n-4} \tag{30}$$

so that $c_{n-m,m}$ is dimensionless. Notice that if we define $\mathrm{t} = Ht$ and $V(\phi) = H^4 V(\varphi_+, \varphi_-)$, then the action, $S \supset \int d\mathrm{t} d^3x [aH]^3 V(\varphi_+, \varphi_-)$ implements the explicit power counting in $[aH]$ where φ_+ and φ_- carry dimensions α and $\beta, \mathbf{x}$ is dimension -1, and t is dimension 0 . We can express this more transparently by expanding in the dimension of the operator, $\Delta_{n,m} = (n-m)\alpha + m\beta$, so that our potential becomes

因此 $c_{n-m,m}$ 是无量纲的。注意，如果我们定义 $\mathrm{t} = Ht$ 和 $V(\phi) = H^4 V(\varphi_+, \varphi_-)$，则作用量 $S \supset \int d\mathrm{t} d^3x [aH]^3 V(\varphi_+, \varphi_-)$ 在 $[aH]$ 中实现了显式幂次计数: 其中 φ_+ 和 φ_- 的量纲为 α，$\beta, \mathbf{x}$ 的量纲为-1，t 的量纲为 0。我们可以通过按算符的量纲展开让表达更清晰，即 $\Delta_{n,m} = (n-m)\alpha + m\beta$，因此势可以写为

$$\begin{aligned} S_{n.m} &= \int d\mathbf{t} d^3x [aH]^{3-(n-m)\alpha - m\beta} \frac{c_{n-m,m}}{(n-m)!m!} \varphi_+^{n-m} \varphi_-^m (\mathbf{x}) \\ &= \int d\mathrm{t} d^3x \frac{1}{\Lambda^{3-\Delta_{n,m}}} \frac{c_{n-m,m}}{(n-m)!m!} \varphi_+^{n-m} \varphi_-^m (\mathbf{x}) \end{aligned} \tag{31}$$

where $\Lambda(t) = a(t) H$ is the (time-dependent) UV cutoff, as desired.

其中 $\Lambda(t) = a(t) H$ 正如所求，是 (含时的) 紫外截断。

So far, we have described SdSET from the top down by expressing the short-distance field ϕ in terms of $\varphi_\pm$. However, like any good EFT, we can derive all the same result from the bottom up using only the degrees of freedom below UV cutoff $\Lambda(t), \varphi_\pm$, the symmetries, and the power counting rules. The special relations between $c_{n-m,m}$ and $c_{n-m',m'}$ that arise from matching to the UV potential, Equation (30), occur in the EFT due to an additional symmetry, reparameterization invariance (RPI):

到目前为止，我们已经通过从上到下的方式，将短距离场 ϕ 用 $\varphi_\pm$ 表示，描述了 SdSET。但和任何成熟的有效场论一样，我们仅用紫外截断 $\Lambda(t)$, $\varphi_\pm$ 以下的自由度、对称性和幂次计数规则，就可以从下到上推导出完全相同的结果。通过匹配紫外势 (式 (30)) 得到的 $c_{n-m,m}$ 和 $c_{n-m',m'}$ 之间的特殊关系，在该有效场论中源于一个额外对称性: 重参数化不变性 (RPI):

$$\varphi_+ \to \varphi_+ + \varepsilon[aH]^{\alpha-\beta}\varphi_- \tag{32}$$

$$\varphi_- \to (1-\varepsilon)\,\varphi_- \tag{33}$$

This is a symmetry of the low-energy theory that keeps the original field, ϕ , fixed. In addition to symmetries, the rules of EFT also permit us to change variables to remove (redundant) terms in the action. Interestingly, in SdSET, there are a number of changes of variables that are not field redefinitions of the UV theory. Concretely, we can redefine φ_+ and φ_- independently to remove a number of couplings from the action, yet in the original description one cannot express this as a field redefinition of ϕ itself. This additional flexibility is what allows SdSET to simplify and clarify a number of features of physics in dS.

这是低能理论的一个对称性，它保持原场 ϕ 固定。除对称性外，有效场论的规则还允许我们通过变量替换消去作用量中的 (冗余) 项。有意思的是，在 SdSET 中，存在许多不属于紫外理论场重定义的变量替换。具体来说，我们可以独立重定义 φ_+ 和 φ_- ，从作用量中消去多个耦合，但在原描述中这无法表示为 ϕ 本身的场重定义。这种额外的灵活性正是 SdSET 能够简化并阐明德西特空间中许多物理性质的原因。

Naively, power counting suggests that φ_+^n is a relevant operator when $n\alpha < 3$, which would imply that the corrections to the scalar field dynamics grow like powers of $[aH]$. If true, scalar fields with a wide range of masses would be strongly coupled on super-horizon distances in dS. However, from explicit diagrammatic arguments, it has long been known that perturbative corrections grow at most logarithmically in aH [104, 105] . We can understand the absence of power-law growth in the UV description by writing the in-in correlator in the commutator form, Equation (11) with $\varepsilon = 0$,

朴素来看，幂次计数表明当 $n\alpha < 3$ 时，φ_+^n 是相关算符，这意味着对标量场动力学的修正会随 $[aH]$ 的幂次增长。如果这是真的，那么在德西特空间的超视界尺度上，大质量范围内的标量场都会是强耦合的。但通过显式图论论证，人们很早就知道微扰修正最多随 aH [104, 105] 对数增长。我们可以通过将入-入关联函数写为对易子形式 (即式 (11)，其中 $\varepsilon = 0$)，来理解紫外描述中为什么不存在幂律增长:

$$\begin{aligned}\langle \mathcal{O}(t)\rangle = \sum_{N=0}^{\infty} i^N \int_{-\infty}^{t} \mathrm{d}t_N \int_{-\infty}^{t_N} \mathrm{d}t_{N-1} \cdots \int_{-\infty}^{t_2} \mathrm{d}t_1 \\ \times \left\langle \left[H_{\text{int}}(t_1), \left[H_{\text{int}}(t_2), \cdots \left[H_{\text{int}}(t_N), \mathcal{O}^{\text{int}}(t)\right]\cdots\right]\right]\right\rangle\end{aligned} \tag{34}$$

Because every insertion of the interaction is associated with a commutator, $[\dot{\phi}, \phi] \propto a^{-3}$, the overall power of a^3 in the measure of integration is always cancelled so that no positive power of a appears alone. This argument is straightforward at tree level but is technically complicated for general loop diagrams.

因为每次相互作用插入都对应一个对易子 $[\dot{\phi},\phi] \propto a^{-3}$ ，积分测度中 a^3 的总幂次总会被抵消，因此不会单独出现 a 的正幂次。这个论证在树图层面很直接，但对一般圈图来说技术上很复杂。

In the EFT description, the absence of power-law growth to all-loop order arises simply because φ_+^n is a redundant operator: it can be removed by a field definition

在有效场论描述中，全阶都不存在幂律增长，原因很简单: φ_+^n 是一个冗余算符，它可以通过场定义消去

$$\varphi_- \to \varphi_- + \frac{c_{n,0}}{9\,(n-1)!}[aH]^{3-(n-1)\alpha}\varphi_+^{n-1}. \tag{35}$$

This is precisely the kind of field redefinition that exists only the EFT and not the full theory, thus explaining why a complicated result of the full theory can be made trivial in the EFT. As stated above, this does not replace our need to perform the full UV calculations but is one of several all-order results that is translated into a simple power counting argument in EFT.

这正是仅存在于有效场论 (EFT) 而非完整理论中的场重定义，由此解释了为何完整理论的复杂结果能在 EFT 中被简化。如上所述，这并未取代我们进行完整紫外计算的需求，它仅是若干全阶结果之一，可在 EFT 中转化为简单的幂计数论证。

After removing redundant operators, the leading interaction by power counting is $c_{n-1,1}$ which is associated with a dimension $\Delta_{n-1,1} = 3 + (n-2)\,\alpha$ operator. This operator is irrelevant $\left(\Delta_{n-1,1} > 3\right)$ for all $\alpha > 0$, and therefore, we see that a typical (massive) scalar in dS is a free field in the IR. Similarly, derivative interactions have dimensions $\Delta_\partial \geq 5$ for all $\alpha \geq 0$ and are always irrelevant. This leaves the case of massless scalar field, $\alpha = 0$, as the unique situation where there could be nontrivial long-distance dynamics on superhorizon scales.

移除冗余算符后，通过幂计数得到的领头阶相互作用是 $c_{n-1,1}$ ，它对应一个维度为 $\Delta_{n-1,1} = 3 + (n-2)\,\alpha$ 的算符。对于所有 $\alpha > 0$ ，该算符都是无关 $\left(\Delta_{n-1,1} > 3\right)$ 算符，因此我们可以看到，德西特空间中典型的 (有质量) 标量场在红外区是自由场。类似地，导数相互作用对所有 $\alpha \geq 0$ 的维度均为 $\Delta_\partial \geq 5$ ，也始终是无关算符。因此，只有无质量标量场 $\alpha = 0$ 这一种情况，会在超视界尺度上存在非平庸长程动力学。

The precise field redefinition needed to remove redundant operators is important for matching, as it modifies the effective potential of the EFT relative to the potential of the UV theory. Specifically, if we start from $V(\phi) = \lambda\phi^4/4\,!$ and $\alpha = 0$, then repeated application of this field redefinition to remove all the $c_{n,0}$ terms generates an effective potential in the EFT [33],

移除冗余算符所需的精确场重定义对匹配十分重要，因为相对于紫外理论的势，它会改变 EFT 的有效势。具体来说，如果我们从 $V(\phi) = \lambda\phi^4/4$ 和 $\alpha = 0$ 出发，反复应用该场重定义移除所有 $c_{n,0}$ 项，就会在 EFT 中生成一个有效势 [33],

$$V_{\text{eff}}(\varphi_+, \varphi_-) \supset \frac{\lambda}{3!}\varphi_- \left(\varphi_+^3 + \frac{\lambda}{18}\varphi_+^5 + \frac{\lambda^2}{162}\varphi_+^7 + \dots\right). \tag{36}$$

These higher-order terms are precisely those necessary to match the behavior of the UV theory. The first few terms in this effective potential also reproduce perturbative calculations of the effective potential of light fields calculated using wavefunction [50] or static patch techniques [70].

这些高阶项恰好是匹配紫外理论行为所必需的。该有效势的前几项也能重现利用波函数 [50] 或静态补丁技术 [70] 计算得到的轻场有效势的微扰结果。

In addition to the interactions in the EFT itself, interactions in the UV theory modify the initial conditions of the EFT, in the form of time-independent corrections (At zeroth order in the EFT, $\dot{\varphi}_\pm = 0$, we can always add time-independent initial statistics while remaining consistent the dynamics of the EFT.) to statistics of φ_+ . These must be matched to the full theory as they are not predictions of the EFT itself. For example, if we add a $\left(\partial_\mu \phi \partial^\mu \phi\right)^2$ interaction to the UV theory, as in Equation (13), then we must match the ϕ correlator in Equation (14) with the φ_+ initial conditions such that

除了 EFT 本身的相互作用外，紫外理论的相互作用还会以不随时间改变的修正形式 (在 EFT 的零阶 $\dot{\varphi}_\pm = 0$ 下，我们总可以添加不随时间改变的初始统计量，且仍与 EFT 的动力学自洽) 修正 φ_+ 统计量的初始条件。这些修正必须通过与完整理论匹配得到，因为它们并非 EFT 本身就能预言。例如，如果我们像式 (13) 那样给紫外理论添加一个 $\left(\partial_\mu \phi \partial^\mu \phi\right)^2$ 相互作用，我们就必须让式 (14) 的 ϕ 关联函数与 φ_+ 初始条件匹配，使得

$$\langle \varphi_+ (\mathbf{k}_1, t) \dots \varphi_+ (\mathbf{k}_4, t) \rangle \supset \frac{H^4}{\Lambda^4} \frac{P_6 (\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3 \mathbf{k}_4)}{16 (k_1 k_2 k_3 k_4)^3 k_t^5} (2\pi)^3 \delta \left(\sum \mathbf{k}_i\right). \tag{37}$$

In this precise sense, the SdSET does not replace the need to perform the UV calculations, as the statistics determined by the UV theory. However, the key observation is that the initial conditions are time-independent and thus do not influence the time evolution of the fundamental fields, $\varphi_\pm (\mathbf{k}, \mathrm{t})$.

严格来说，软德西 itter 有效理论 (SdSET) 并未取代紫外计算的必要性，因为初始统计量由紫外理论决定。但关键结论是，初始条件不随时间改变，因此不会影响基本场 $\varphi_\pm (\mathbf{k}, \mathrm{t})$ 的时间演化。

In the previous section, we saw that there can be nontrivial evolution of the superhorizon modes, in the form of anomalous dimensions. This is important for both massive and massless fields, as we saw they must arise for conformal mass scalars with $m^2 = 2H^2$. One might wonder how this is consistent with the claim that $\alpha > 0$ has only irrelevant interactions. The reason it is consistent is that composite operators require us to integrate over the statistical correlators, including the initial conditions

在前一节中我们看到，超视界模式可以反常维度的形式存在非平庸演化。这对有质量场和无质量场都很重要，我们已经知道，对于共形质量标量，当 $m^2 = 2H^2$ 时这种演化必然出现。有人可能会疑惑，这和“$\alpha > 0$ 仅包含无关相互作用”的论断如何自洽？二者自洽的原因在于，复合算符要求我们对统计关联函数 (包括初始条件) 做积分。

$$\langle \varphi_+^n (\mathbf{x}, t) \dots \rangle = \int \left(\prod_{i=1}^{n} \frac{d^3 \mathbf{k}_i}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}} \right) \langle \varphi_+ (\mathbf{k}_1, t) \dots \varphi_+ (\mathbf{k}_n, t) \dots \rangle \tag{38}$$

The correlation function on the right-hand side includes the contributions from the initial conditions that are determined by matching. Since the only scale in the problem is $[aH]$, if the k -integrals are logarithmically divergent, they will produce factors of $\log k_i/[aH]$. From the EFT point of view, these are UV divergences and signal the need for (dynamical) RG flow. This is a standard phenomena in EFT where, in the right variables, IR divergences of the microscopic description are replaced by UV divergences of the EFT [30]. The resulting dynamical RG in SdSET gives us an anomalous dimension for φ_+^n , like the one we found in Equation (19).

右侧的关联函数包含由匹配确定的初始条件贡献。由于问题中唯一的能标是 $[aH]$ ，若 k 积分存在对数发散，就会产生因子 $\log k_i/[aH]$ 。从有效场论的角度来看，这些是紫外发散，表明需要 (动力学) 重整化群流。这是有效场论中的标准现象: 在合适的变量下，微观描述的红外发散会被有效场论的紫外发散取代 [30]。软德西特有效理论中最终得到的动力学重整化群会为 φ_+^n 给出反常维数，和我们在式 (19) 中得到的结果一致。

Note that the anomalous dimensions are only generated in this way for the composite operators. In contrast, the dimensions of $\varphi_\pm$ are not altered in the EFT, except through interactions in the EFT itself. The reason is that the dimensions α and β are determined by matching and there is no meaningful notion of an anomalous dimension of $\varphi_\pm$. Instead, if such a dimension is generated in the UV (including by a perturbative shift of the effective mass), it is simply absorbed into α by matching.

注意，只有复合算符才会通过这种方式产生反常维数。相比之下，$\varphi_\pm$ 的维数在有效场论中不会被改变，除非通过有效场论自身的相互作用改变。原因在于，维数 α 和 β 由匹配确定，不存在关于 $\varphi_\pm$ 反常维数的有意义定义。反之，如果紫外区产生了这样的维数 (包括有效质量的微扰偏移)，它只会通过匹配被吸收到 α 中。

Light Scalars

轻标量场

Light scalars with $m \ll H$ have long been known to present a significant challenge in de Sitter space. Because their power spectrum is scale-invariant, Equation (9), momentum integrals typically diverge at $k = 0$. In addition, even tree-level interactions can give rise to secular growth that needs to be resummed. The framework known as stochastic inflation [10, 90, 100] has long been known to be free of these problems, and it has been suggested that it is responsible for resolving these IR issues [42,75,82,83]. Stochastic inflation translates the freeze-out of each mode of a fundamental scalar field ϕ at horizon crossing to a step in a random walk describing the local value of the field. This intuition gives rise to a Fokker-Planck equation for the classical probability distribution for $\phi, P(\phi, t)$, in terms of H and the scalar's potential $V(\phi)$,

人们早已知道，满足 $m \ll H$ 的轻标量场在德西特空间中是一个重大难题。由于其功率谱是标度不变的 (见式 (9))，动量积分通常在 $k = 0$ 处发散。此外，即使是树图相互作用也会产生长期增长，需要重求和。人们早就发现随机暴涨框架 [10, 90, 100] 不存在这些问题，且已有观点认为该框架可以解决这些红外问题 [42,75,82,83]。随机暴涨将基本标量场 ϕ 各模式在视界穿越时的冻出，转化为描述场局域取值的随机游走中的一步。这一思路导出了针对 $\phi, P(\phi, t)$ 经典概率分布的福克-普朗克方程，该方程用 H 和标量势 $V(\phi)$ 表示为，

$$\frac{\partial}{\partial t} P(\phi, t) = \frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \phi^2} P(\phi, t) + \frac{1}{3H} \frac{\partial}{\partial \phi} \left[V'(\phi) P(\phi, t) \right]. \tag{39}$$

The first term represents of (Gaussian) quantum noise at horizon crossing, while the second term is that classical drift. This equation has been derived from a number of perspectives and has been extended to multi-field models to include nontrivial field-space geometry [74]. Yet, we have seen that the effective potential $V_{\text{eff}}(\varphi_+, \varphi_-)$, Equation (36), does not match $V(\phi)$ beyond leading order in λ , and therefore it is unclear if or how such corrections should be included in the stochastic framework. We would like to understand from EFT power counting how stochastic inflation arises, its regime of applicability, and how to calculate corrections [43,66,99].

第一项代表视界穿越处的 (高斯) 量子噪声，第二项则是经典漂移。该方程已经从多个角度推导得到，并被推广到多场模型，纳入了非平凡场空间几何 [74]。但我们已经看到，有效势 $V_{\text{eff}}(\varphi_+, \varphi_-)$ (式 (36)) 在超出领头阶 λ 时与 $V(\phi)$ 不匹配，因此这类修正是否应该、又该如何纳入随机框架尚不明确。我们希望从有效场论幂计数出发，理解随机暴涨的起源、适用范围以及修正的计算方法 [43,66,99]。

From the action of SdSET, we can observe that theories with massless scalars ($\alpha \to 0$) are the unique situation where the superhorizon theory can have nontrivial IR dynamics. By power counting, potential interactions are marginal when $\alpha \to 0$ and thus do not decouple at late times. In addition, the massless limit is important because all the composite operators of the form φ_+^n have the same dimension in the limit $\alpha \to 0, \Delta_{n,0} = n\alpha \to 0$ for all n . As operators with the same dimension can mix under RG, this infinite tower of operators can mix in the massless limit, introducing a highly nontrivial RG flow.

从德西特有效场论作用量出发, 我们可以发现, 包含无质量标量场 ($\alpha \to 0$) 的理论是超视界理论能够拥有非平凡红外动力学的唯一情形。通过幂计数可知, 当满足 $\alpha \to 0$ 时, 势相互作用是边缘的, 因此不会在晚期退耦。此外, 无质量极限十分重要, 因为所有形如 φ_+^n 的复合算符在 $\alpha \to 0, \Delta_{n,0} = n\alpha \to 0$ 极限下对任意 n 都具有相同维度。由于维度相同的算符会在重整化群下混合，这一无穷算符塔可以在无质量极限下混合，形成高度非平凡的重整化群流。

The nontrivial mixing of scalar operators is already present in the free theory, due to the scalar invariance of the power spectrum in Equation (9). We will regulate this divergence by analytic continuation in α and then taking $\alpha \to 0$. From the Wick contraction of two fields, $\alpha \to 0$ gives

由于式 (9) 中功率谱具有标量不变性，标量算符的非平凡混合已经在自由理论中出现。我们将通过对 α 做解析延拓来规制这个发散，随后再取 $\alpha \to 0$ 。对两个场做威克收缩后，$\alpha \to 0$ 给出

$$\langle \varphi_+^2(\mathbf{x}) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{C_\alpha^2}{k^{3-2\alpha}} \to [aH]^{2\alpha} \int \frac{d^3k}{(2\pi)^3} \frac{C_\alpha}{(k^2+m^2)^{3/2-\alpha}} \tag{40}$$

$$= \frac{1}{2\alpha} - \log(k/[aH]) + \ldots \tag{41}$$

The original integral is both UV and IR divergent, but we regulate the IR, introducing an artificial m^2 to isolate the UV divergence. Including the RG flow from the UV to the IR softens the IR behavior and allows us to take $m^2 \to 0$. After implementing the RG, we will no longer need the IR regulator as RG resolves in the long-distance behavior.

原始积分同时存在紫外和红外发散，我们规制红外发散时，引入人工 m^2 来分离紫外发散。纳入从紫外到红外的重整化群流会软化红外行为，使我们可以取 $m^2 \to 0$ 。完成重整化后，我们不再需要红外规制，因为重整化解决了长距离行为的问题。

Combining the divergences from this Wick contraction and the classical evolution of the massless fields, one finds the Callan-Symmanzik equation for this mixing [16,17]:

结合该威克收缩的发散和无质量场的经典演化，可以得到描述这种混合的卡伦-西曼齐克方程 [16,17]:

$$\frac{\partial}{\partial t}\left\langle\varphi_{+}^{n}(\mathbf{x})\ldots\right\rangle=\frac{n(n-1)}{8\pi^{2}}\left\langle\varphi_{+}^{n-2}(\mathbf{x})\ldots\right\rangle-\frac{n}{3}\sum_{m>1}\frac{c_{m,1}}{m!}\left\langle\varphi_{+}^{n-1}(\mathbf{x})\varphi_{+}^{m}(\mathbf{x})\ldots\right\rangle.$$

(42)

Note that neither derivatives of φ_+ nor powers of φ_- appear in this equation because they are at least dimension two and three, respectively. Given that the nonzero commutator in SdSET is $[\varphi_+,\varphi_-]$, quantum-mechanical effects are formally irrelevant to do not impact superhorizon physics (whether or not there is decoherence).

注意该方程中既不出现 φ_+ 的导数，也不出现 φ_- 的幂次，因为它们分别至少是二维和三维算符。鉴于德西特有效场论中的非零对易子为 $[\varphi_+,\varphi_-]$ ，量子效应形式上是无关的，不会影响超视界物理 (无论是否存在退相干)。

Now, suppose we consider the case of a massless ϕ with potential $V(\phi)=\lambda\phi^4/4!$ as the UV theory. SdSET must inherit the $\phi\to-\phi$ symmetry, but we would otherwise expect all possible mixings allowed by this symmetry to arise. These corrections can also be computed directly in the UV theory, but again we use SdSET with φ_+ to make the power counting obvious. Specifically, we know the mixing of operators of the same dimension is not limited in any obvious way, and we should expect a sum of the form

现在，假设我们考虑紫外理论中存在一个无质量 ϕ ，势能为 $V(\phi)=\lambda\phi^4/4!$ 的情形。SdSET 必须继承 $\phi\to-\phi$ 对称性，但除此之外我们预期该对称性允许的所有可能混合都会出现。这些修正也可以直接在紫外理论中计算，但我们再次结合 φ_+ 使用 SdSET，以使幂次计数更加清晰。具体而言，我们知道相同维度算符的混合没有任何明显限制，因此我们预期会出现如下形式的求和

$$\frac{\partial}{\partial t}\left\langle\varphi_{+}^{n}\right\rangle=-\frac{n}{3}\sum_{m>1}^{\text{odd}}\frac{c_{m,1}}{m!}\left\langle\varphi_{+}^{n+m-1}\right\rangle+\binom{n}{2}\sum_{m=0}^{\infty}b_{m}\left\langle\varphi_{+}^{n+2m-2}\right\rangle$$
$$-\binom{n}{3}\sum_{m=0}^{\infty}d_{m}\left\langle\varphi_{+}^{n+2m-2}\right\rangle+\binom{n}{4}\sum_{m=0}^{\infty}e_{m}\left\langle\varphi_{+}^{n+2m-4}\right\rangle+\ldots \tag{43}$$

Interestingly, the information encoded by these equations can be written as a single equation for the probability distribution of φ_+ ,

有趣的是，这些方程编码的信息可以写成一个关于 φ_+ 概率分布的单一方程,

$$\frac{\partial}{\partial t}P(\varphi_{+},t)=\frac{1}{3}\frac{\partial}{\partial\varphi_{+}}\left[\partial_{\varphi_{-}}V_{\text{eff}}(\varphi_{+},\varphi_{-})\big|_{\varphi_{-}=0}P(\varphi_{+},t)\right]$$
$$+\frac{\partial^{2}}{\partial\varphi_{+}^{2}}\left[\sum_{m=0}^{\infty}\frac{b_{m}}{2!}\varphi_{+}^{2m}P(\varphi_{+},t)\right]+\frac{\partial^{3}}{\partial\varphi_{+}^{3}}\left(\varphi_{+}\sum_{m=0}^{\infty}\frac{d_{m}}{3!}\varphi_{+}^{2m}P(\varphi_{+},t)\right)$$

$$+\frac{\partial^4}{\partial\varphi_+^4}\left(\sum_{m=0}^{\infty}\frac{e_m}{4!}\varphi_+^{2m}P(\varphi_+,t)\right)+\ldots \tag{44}$$

This infinite series of terms is typical of a general Markovian process where we can write the time evolution as

> 这种无穷项级数是一般马尔可夫过程的典型特征，我们可以将其时间演化写为

$$\frac{\partial}{\partial \mathrm{t}}P(\varphi_+,\mathrm{t})=\int d\varphi'_+\left(W(\varphi_+\mid\varphi'_+)P(\varphi'_+,\mathrm{t})-W(\varphi'_+\mid\varphi_+)P(\varphi_+,\mathrm{t})\right) \tag{45}$$

where $W(\varphi_+\mid\varphi'_+)$ is the transition amplitude for a jump from φ'_+ to φ_+ . The derivative expansion in Equation (44) is the result of expanding in $\varphi_+-\varphi'_+$, which is known as the Kramers-Moyal expansion. The coefficients b_m characterized the variance of the Gaussian noise as a function of φ_+ , while the higher-order terms like d_m and e_m are the non-Gaussian moments of the transition amplitudes.

> 其中 $W(\varphi_+\mid\varphi'_+)$ 是从 φ'_+ 跃迁至 φ_+ 的跃迁振幅。式 (44) 中的导数展开是对 $\varphi_+-\varphi'_+$ 展开的结果，这就是熟知的克拉默斯-莫约尔展开。系数 b_m 表征了高斯噪声方差关于 φ_+ 的函数，而 d_m 和 e_m 这类高阶项则是跃迁振幅的非高斯矩。

These types of corrections to the stochastic framework can be derived in many ways [26, 50, 69, 70] , all of them arising from integrating out modes with momenta $p > aH$. Several of these approaches directly integrate out our φ_- by focusing only on the nearly constant mode [26, 69, 70] . We cannot write a first-order action for φ_+ without φ_- , but after integrating out φ_- , one can still arrive at an effective equation of motion like Equation (44) using the language of open EFT [25]. From the perspective of SdSET, we would expect this procedure only works when $\alpha\approx 0$ so that φ_- and φ_+ don’t mix. This description has the advantage of making the connection to the thermal behavior of the static patch more explicit [7, 69, 70]. Alternatively, one can work with the wavefunction of the universe as an intermediate step [50], which is advantageous for isolating the origin of the logarithmic terms [6]. Similar results can be derived directly from the in-in expression by diagrammatic arguments [16, 17]. In a certain sense, all these approaches behave as EFTs in that they remove the subhorizon physics.

> 这类对随机框架的修正可以通过多种方法推导 [26, 50, 69, 70] ，所有修正都来源于积去动量为 $p > aH$ 的模式。其中若干方法仅关注近常数模式 [26, 69, 70] ，直接积去我们的 φ_- 。若没有 φ_- 我们无法写出 φ_+ 的一阶作用量，但在积去 φ_- 后，我们仍然可以利用开 EFT 的语言得到式 (44) 这类有效运动方程 [25]。从 SdSET 的角度来看，我们预期该过程仅在 $\alpha\approx 0$ 时成立，此时 φ_- 和 φ_+ 不发生混合。这种描述的优势是可以更清晰地建立与静态补丁热行为的联系 [7, 69, 70]。另外，也可以将宇宙波函数作为中间步骤进行计算 [50]，这有利于分离对数项的起源 [6]。我们还可以通过图论论证直接从 in-in 表达式得到相似结果 [16, 17]。从某种意义上说，所有这些方法都等价于 EFT，即它们都移除了亚视界物理。

The concrete advantage of SdSET is that it reduces the problem to calculating the matrix of anomalous dimensions from scaleless loop integrals. In this precise sense, calculating the coefficients of the Kramers-Moyal expansion is essentially identical to finding the dimensions of operators at the Wilson-Fisher fixed point in $d=4-\varepsilon$ dimensions. The first higher derivative term, e.g., d_0 in $\lambda\phi^4$, can only be calculated using this method, which at the appropriate order in λ gives the corrected equation [33]

SdSET 的具体优势在于，它将问题约化为从无标度圈积分计算反常维度矩阵。严格来说，计算克拉默斯-莫约尔展开的系数本质上等价于求解 $d = 4 - \varepsilon$ 维中威尔逊-费希尔不动点处算符的维度。第一个高阶导数项，例如 $\lambda\phi^4$ 中的 d_0 ，只能通过该方法计算，在 λ 的合适阶次下可以得到修正后的方程 [33]

$$\frac{\partial}{\partial t}P\left(\overline{\varphi}_+,t\right)=\frac{1}{3}\frac{\partial}{\partial\overline{\varphi}_+}\left[V'_{\text{eff}}\left(\overline{\varphi}_+\right)P\left(\overline{\varphi}_+,t\right)\right]+\frac{1}{8\pi^2}\frac{\partial^2}{\partial\overline{\varphi}_+^2}P\left(\overline{\varphi}_+,t\right)$$

$$+\frac{\lambda_{\text{eff}}}{1152\pi^2}\frac{\partial^3}{\partial\overline{\varphi}_+^3}\left(\overline{\varphi}_+P\left(b\overline{\varphi}_+,t\right)\right) \tag{46}$$

$$V'_{\text{eff}}=\frac{\lambda_{\text{eff}}}{3!}\left(\overline{\varphi}_+^3+\frac{\lambda_{\text{eff}}}{18}\overline{\varphi}_+^5+\frac{\lambda_{\text{eff}}^2}{162}\overline{\varphi}_+^7+\ldots\right). \tag{47}$$

where $\lambda \to \lambda_{\text{eff}}$ and $\varphi_+ \to \overline{\varphi}_+$ were redefined to remove b_1 and b_2 terms.

其中我们重新定义了 $\lambda \to \lambda_{\text{eff}}$ 和 $\varphi_+ \to \overline{\varphi}_+$ ，以移除 b_1 项和 b_2 项。

Despite the advantages of the SdSET approach, it remains tied to the UV calculation via the matching of initial conditions. Other approaches may be able to circumvent this technical requirement while maintaining power counting. For example, Mellin space makes loop integrals scaleless and can simplify some of the technical complications of regulating the UV description [78]. In addition, objects like the wavefunction of the universe [54, 56] might be more natural starting points for non-perturbative dynamical RG [28,51], in analogy with the exact RG results from the path integral. SdSET is a continuum EFT, in the sense of [47], which has technical advantages for concrete calculations, but lacks the conceptual and non-perturbative advantages of Wilsonian EFT.

尽管 SdSET 方法具备优势，但它仍需通过初始条件匹配与紫外计算绑定。其他方法或许能在保留幂次计数的同时避开这一技术要求。例如，梅林空间让圈积分无量纲，可以简化紫外描述正规化中的部分技术难题 [78]。此外，类比路径积分的精确重整化群结果，宇宙波函数 [54, 56] 这类对象或许是非微扰动力学重整化群更自然的出发点。SdSET 属于文献 [47] 意义上的连续有效场论，它在具体计算中具备技术优势，但缺乏威尔森有效场论的概念性与非微扰优势。

Given the expansion in derivatives with respect to φ_+ in our effective Fokker-Planck equation, Equation (46), it is reasonable to wonder what parameter controls the size of the higher derivative terms. To gain intuition, let us start by truncating the equations at two derivatives and linear order in λ_{eff} , namely, the Fokker-Planck equation:

鉴于我们的有效福克-普朗克方程 (式 (46)) 中是相对于 φ_+ 的导数展开，我们自然会好奇是哪个参数控制高阶导数项的大小。为建立直观认识，我们先将方程截断到两阶导数和 λ_{eff} 一阶线性项，即得到如下福克-普朗克方程:

$$\frac{\partial}{\partial \mathrm{t}}P\left(\overline{\varphi}_+,\mathrm{t}\right)=\frac{1}{3}\frac{\partial}{\partial\overline{\varphi}_+}\left[\frac{\lambda_{\text{eff}}}{3!}\varphi_+^3P\left(\overline{\varphi}_+,\mathrm{t}\right)\right]+\frac{1}{8\pi^2}\frac{\partial^2}{\partial\overline{\varphi}_+^2}P\left(\overline{\varphi}_+,\mathrm{t}\right). \tag{48}$$

We can calculate the equilibrium probability distribution [92], $\dot{P}^{\text{FP}}_{\text{eq}}\left(\varphi_+\right)=0$, by direct integration to find

我们可以通过直接积分计算平衡概率分布 [92]，即 $\dot{P}_{\mathrm{eq}}^{\mathrm{FP}}(\varphi_+) = 0$ ，得到

$$P_{\mathrm{eq}}^{\mathrm{FP}}(\varphi_+) = \exp\left(-\frac{\pi^2}{9}\lambda_{\mathrm{eff}}\varphi_+^4\right). \tag{49}$$

Since the probability is of order one up to $|\varphi_+| \sim \lambda_{\mathrm{eff}}^{-1/4}$, we see that the counting in $\lambda_{\mathrm{eff}} \ll 1$ is modified by large values of φ_+ in equilibrium. Using the power counting $\varphi_+ \sim \lambda_{\mathrm{eff}}^{-1/4}$, the leading order behavior (LO) of Equation (46), $\mathcal{O}\left(\lambda_{\mathrm{eff}}^{1/2}\right)$, is given by standard stochastic inflation, as in Equation (48). At next-to-leading order (NLO), $\mathcal{O}(\lambda_{\mathrm{eff}})$, only the φ_+^5 term in $V'_{\mathrm{eff}}(\varphi_+)$ contributes, with the remaining terms being next-to-next-to-leading order (NNLO), $\mathcal{O}\left(\lambda_{\mathrm{eff}}^{3/2}\right)$. We can write the equilibrium probability distribution as a similar expansion in λ_{eff} , $P_{\mathrm{eq}} = CP_{\mathrm{LO}}(\varphi_+)P_{\mathrm{NLO}}(\varphi_+)P_{\mathrm{NNLO}}(\varphi_+)$, such that the solution to NNLO is [33]

由于概率在 $|\varphi_+| \sim \lambda_{\mathrm{eff}}^{-1/4}$ 量级以内为 O(1), 我们可以看到, 平衡态下 φ_+ 的大值会修改 $\lambda_{\mathrm{eff}} \ll 1$ 中的计数。采用幂次计数 $\varphi_+ \sim \lambda_{\mathrm{eff}}^{-1/4}$, 式 (46) 的领头阶 (LO) 行为 $\mathcal{O}\left(\lambda_{\mathrm{eff}}^{1/2}\right)$ 由标准随机通货膨胀给出, 即式 (48)。在次领头阶 (NLO) $\mathcal{O}(\lambda_{\mathrm{eff}})$, 只有 $V'_{\mathrm{eff}}(\varphi_+)$ 中的 φ_+^5 项有贡献, 其余项均为次次领头阶 (NNLO) $\mathcal{O}\left(\lambda_{\mathrm{eff}}^{3/2}\right)$。我们可以将平衡概率分布写为 λ_{eff} 下的类似展开 $P_{\mathrm{eq}} = CP_{\mathrm{LO}}(\varphi_+)P_{\mathrm{NLO}}(\varphi_+)P_{\mathrm{NNLO}}(\varphi_+)$, 因此 NNLO 的解为 [33]

$$P_{\mathrm{LO}} = \exp\left(-\frac{\pi^2}{9}\lambda_{\mathrm{eff}}\varphi_+^4\right) \tag{50}$$

$$P_{\mathrm{NLO}} = \exp\left(-\frac{\pi^2}{243}\lambda_{\mathrm{eff}}^2\varphi_+^6\right) \tag{51}$$

$$P_{\mathrm{NNLO}} = \exp\left(\frac{5}{10368}\lambda_{\mathrm{eff}}^2\varphi_+^4 - \frac{17\pi^2}{46656}\lambda_{\mathrm{eff}}^3\varphi_+^8\right). \tag{52}$$

The same power counting can be applied to the relaxation eigenvalues,

同样的幂次计数也可应用于弛豫本征值,

$$\frac{d}{dt}P_i(\varphi) = -\Lambda_i P_i(\varphi) \tag{53}$$

which have been similarly calculated to NLO [50,70] and NNLO [33].

其已被类似地计算到 NLO[50,70] 和 NNLO[33]。

The key take-away is that from a number of distinct perspectives [6, 16, 17, 26, 31, 33, 50, 69, 70] , the IR divergences and secular growth of the massless scalar fields in FRW slicing of dS are now understood. EFT trades the bad IR behavior for UV divergences in the usual sense. The UV divergences give rise to an RG (or, equivalently, stochastic inflation) and show that the theory flows to a nontrivial fixed point (equilibrium distribution) where $\varphi \sim \lambda^{-1/4}$. This has long been conjectured as the resolution [42, 75, 82, 83] : stochastic inflation is free of the IR problems of in-in perturbation theory and thus would solve the IR issues if it was equivalent to QFT. What recent works have demonstrated is how to derive this result from QFT and how to calculate corrections to these results to any order in λ . More significantly, as we now understand the origin of these challenges in terms of power counting, it is straightforward to generalize these results to any interacting QFT in de Sitter.

核心结论是，从多个不同视角 [6, 16, 17, 26, 31, 33, 50, 69, 70] 来看，德西特空间 FRW 切片中无质量标量场的红外发散和长期增长现在已经得到理解。有效场论按照常规方式将不良红外行为转换为紫外发散。紫外发散催生了重整化群 (或者等价地说，随机通货膨胀)，并表明理论流向非平庸不动点 (平衡分布)，在该不动点处 $\varphi \sim \lambda^{-1/4}$ 。这一点很早就被猜想为问题的解决方向 [42, 75, 82, 83] : 随机通货膨胀没有进-入扰动理论的红外问题，因此如果它等价于量子场论，就能解决红外问题。近期研究已经证明了如何从量子场论推导出这一结果，以及如何在 λ 中将这些结果修正计算到任意阶。更重要的是，由于我们现在已经从幂次计数的角度理解了这些挑战的起源，因此可以很容易地将这些结果推广到德西特空间中的任意相互作用量子场论。

Dynamical Gravity

动力学引力

Dynamical gravity presents an interesting paradox for pure de Sitter and inflationary backgrounds. It is well-known that backreaction of scalar fluctuations changes the nature of the spacetime; in the most extreme case, this gives rise to eternal inflation [53, 62, 93, 100] where even our qualitative understanding is limited. Yet, in perturbation theory, both the scalar and tensor modes are conserved outside the horizon and are much better behaved than most QFTs on a fixed dS background.

动力学引力为纯德西特背景与暴胀背景带来了一个有趣的佯谬。众所周知，标量涨落的反作用会改变时空的本性；在最极端的情况下，这会催生永恒暴胀 [53, 62, 93, 100] ，我们对该场景甚至连定性理解都十分有限。但在微扰论中，标量模和张量模在视界外都是守恒的，其性质比固定德西特背景上的大多数量子场论都要好得多。

Because the metric fluctuates at the future boundary of de Sitter, one might worry that the lack of a physical observable makes the discussion of correlators of metric fluctuations meaningless [21, 108]. Working instead in an inflationary background helps clarify the role of observables, as the background scalar field defines a reference frame in which to compute well-defined correlators that are relevant to observations in our universe. We can work in a gauge where this background field is homogenous, $\phi(\mathbf{x}, t) = \phi(t)$, and the fluctuations are encoded in the metric,

由于度规会在德西特的未来边界涨落，人们可能会担心，缺乏物理观测量使得讨论度规涨落的关联函数毫无意义 [21, 108]。转而在暴胀背景下工作有助于厘清观测量的作用，因为背景标量场定义了一个参考系，我们可以在其中计算与我们宇宙观测相关的、定义良好的关联函数。我们可以在一个背景场均匀的规范下工作，$\phi(\mathbf{x}, t) = \phi(t)$ ，涨落则被编码在度规中，

$$ds^2 = -N^2 dt^2 + a^2(t) e^{2\zeta(\mathbf{x},t)} \left(e^{\gamma(\mathbf{x},t)}\right)_{ij} \left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right), \tag{54}$$

where ζ is the scalar (adiabatic) mode and γ_{ij} are the tensor modes with $\gamma^i_i = 0$ and $\partial_i \gamma^i{}_j = 0$.

其中 ζ 是标量 (绝热) 模，γ_{ij} 是张量模，满足 $\gamma^i_i = 0$ 和 $\partial_i \gamma^i{}_j = 0$.

Inflation as a framework is far more general than just the rolling of a scalar field. The inflationary epoch itself is characterized by an EFT, the EFT of inflation [29], when three conditions are satisfied

暴胀作为一个理论框架，远比单个标量场的 rolling 过程更具一般性。当满足三个条件时，暴胀时期本身可以用一个有效场论——暴胀有效场论来描述 [29]

- The expansion was nearly exponential so that the Hubble parameter, $H(t) = \dot{a}/a$, was nearly constant, $|\dot{H}| \ll H^2$. This condition allows for the creation of scale-invariant long-wavelength fluctuations.

- 膨胀近似为指数膨胀，因此哈勃参数 $H(t) = \dot{a}/a$ 近似为常数，$|\dot{H}| \ll H^2$ 。该条件催生了标度不变的长波涨落。

- There was a physical clock that defined a preferred time slicing. This condition is necessary to allow inflation to end, starting the hot thermal evolution and allowing the long-wavelength fluctuations to seed structure.

- 存在一个物理时钟定义了优先时间切片。该条件是暴胀能够结束、开启后续热大爆炸演化，并让长波涨落成为结构种子的必要条件。

- The preferred slicing manifests itself in the flat space limit (This condition is needed to distinguish the EFT of inflation from models like solid inflation [41], where the cosmological background is the source of the time dependence.) as an operator that breaks time translation, $\langle \mathcal{O} \rangle \propto t$. Inside the horizon, the fluctuations of the metric are characterized by the Goldstone boson, π , of the time translation breaking, through the relation $\zeta = -H\pi + \mathcal{O}(\pi^2)$.

- 优先切片在平直空间极限下表现为一个破坏时间平移对称性的算符，$\langle \mathcal{O} \rangle \propto t$ (该条件用于将暴胀有效场论与固体暴胀这类模型区分开 [41]，在这类模型中宇宙学背景是时间依赖性的来源)。在视界内部，度规涨落由时间平移破缺的戈德斯通玻色子 π 通过关系 $\zeta = -H\pi + \mathcal{O}(\pi^2)$ 刻画。

A wide variety of models of inflation can be made satisfying these criteria, some of which have already been excluded by precision observations of the CMB [2,3]. The EFT of inflation is particularly useful for calculating the statistics of the fluctuations in ζ as they are simply determined from the action for π on a fixed background (to the required accuracy for current observations). In contrast, for determining the superhorizon behavior of the metric in an inflationary background, ζ itself is the more important variable. For our purposes, we can pick the gauge $\pi = 0$ so that we are working directly with the metric fluctuations but at the cost of limiting the utility of the EFT of Inflation.

有大量暴胀模型都满足这些条件，其中一部分已经被宇宙微波背景的精确观测排除 [2,3]。暴胀有效场论对于计算 ζ 中涨落的统计性质尤其有用，因为这些涨落仅由固定背景下 π 的作用量决定 (达到当前观测要求的精度)。与之相反，要确定暴胀背景下度规的超视界行为，ζ 本身才是更重要的变量。出于我们的研究目的，我们可以选取规范 $\pi = 0$ ，从而直接处理度规涨落，但代价是限制了暴胀有效场论的实用性。

The principle origin of the simplicity of metric fluctuations during inflation is that, after gauge fixing the small diffeormorphisms, the metric remains invariant under an infinite set of large diffeomorphisms that act as symmetries of the long-distance theory [58]. The simplest such symmetry is a constant rescaling of coordinates, $\mathbf{x} \to e^{-\lambda}\mathbf{x}$, that shifts the adiabatic mode by a constant $\zeta \to \zeta - \lambda$. This symmetry implies that a physical long-wavelength adiabatic mode is locally indistinguishable from a change of coordinates up to gradients that vanish as $(k/(aH))^2$. This observation is also essential to our conceptual understanding of inflationary fluctuations, including the separate universes approach to calculating cosmological correlators [81, 101]. In addition, this symmetry requires that a constant mode is necessarily a solution to the equations of motion [103], which is central to the conservation of ζ around any FRW background [81].

暴胀过程中度规涨落性质简单的核心原因是，在对小微分同胚做规范固定后，度规在一组无穷多大微分同胚下仍然保持不变，这些大微分同胚是长程理论的对称性 [58]。其中最简单的对称性是坐标的整体重标度 $\mathbf{x} \to e^{-\lambda}\mathbf{x}$，它会让绝热模整体移动一个常数 $\zeta \to \zeta - \lambda$。该对称性表明，物理长波绝热模在局部上无法和坐标变换区分，差别仅在于随 $(k/(aH))^2$ 趋于零的梯度项。这一观测结果对于我们从概念上理解暴胀涨落十分关键，包括计算宇宙关联函数的分宇宙方法 [81, 101]。此外，该对称性要求常数模必然是运动方程的解 [103]，这对于任意 FRW 背景下 ζ 的守恒性是核心 [81]。

More recently, this set of all such symmetries acting on physical fluctuations has been classified. For the adiabatic mode, the key symmetries that act on ζ are [57]

近年来，作用在物理涨落上的所有这类对称性集合已得到分类。对于绝热模式，作用于 ζ 的核心对称性为 [57]

$$D_{\mathrm{NL}} : \delta\zeta = -1 - \mathbf{x} \cdot \partial_{\mathbf{x}}\zeta \tag{55}$$

$$K^i_{\mathrm{NL}} : \delta\zeta = -2x^i - 2x^i(\mathbf{x} \cdot \partial_{\mathbf{x}}\zeta) + x^2\partial^i\zeta, \tag{56}$$

which form a group of nonlinearly realized conformal transformations, $SO(4,1)$, such that ζ acts like a dilaton. The symmetries acting on the tensors are more complicated but can be written in terms of a large diffeomorphism, $\mathbf{x} \to \mathbf{x} + \xi$, and

它们构成了一个非线性实现的共形变换群，$SO(4,1)$，其中 ζ 的作用相当于 dilaton(dilaton 场/标量场，保留原文术语)。作用在张量上的对称性更复杂，但可以用大微分同胚 $\mathbf{x} \to \mathbf{x} + \xi$ 表示为

$$\delta\gamma_{ij} = \partial_i\xi_j + \partial_j\xi_i \tag{57}$$

where

其中

$$\xi_i^{(M)} = M_{i\ell_1}x^{\ell_1} + \frac{1}{2}M_{i\ell_1\ell_2}x^{\ell_1}x^{\ell_2} + \frac{1}{3!}M_{i\ell_1\ell_2\ell_3}x^{\ell_1}x^{\ell_2}x^{\ell_3} + \dots, \tag{58}$$

and $M_{i\ell_1\dots\ell_n}$ are constant tensors that obey a limited set of constraints [58].

且 $M_{i\ell_1\dots\ell_n}$ 是满足一组有限约束的常张量 [58]。

Our understanding of the long-wavelength behavior has been formalized using this full group of symmetries. Inside correlators, they manifest themselves as ward identities that constraint the soft behavior, including Maldacena's single-field consistency condition [63],

我们利用这个完整对称群将长波长行为的研究形式化。在关联函数内部，它们体现为约束软行为的 Ward 恒等式，其中就包括马尔达西那单场一致性条件 [63],

$$\lim_{\mathbf{k}_1\to 0}\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle\to-(n_s-1)P(k_1)P(k_3)(2\pi)^3\delta\left(\sum_i\mathbf{k}_i\right). \tag{59}$$

In short, this expression tells us that long modes decouple from short modes up to an overall coordinate change [37,72]. Acting directly on the operators, symmetries and consistency conditions are also essential in extending the proof of the conservation of ζ to all-loop order. One can use the consistency condition inside individual diagrams to restrict the form of loop corrections [73, 85]. Alternatively, one can use the symmetries to write an operator equation of ζ [11]

简而言之，这个表达式告诉我们，长模式在整体坐标变换之外，和短模式退耦合 [37,72]。直接作用在算符上的对称性和一致性条件，对于将 ζ 守恒性的证明推广到全圈阶也至关重要。我们可以利用单个图内部的一致性条件约束圈修正的形式 [73, 85]。另外，我们也可以利用对称性写出关于 ζ [11] 的算符方程

$$\frac{d}{dt}\zeta(\mathbf{x},t)=\sum_{\Delta}\mathcal{O}_\Delta(\mathbf{x},t)=\frac{1}{[aH]^2}\left(c_1\partial^2\zeta+c_2\partial_i\zeta\partial^i\zeta\right)+\ldots, \tag{60}$$

where the constant $c_{1,2}$, for example, are model-dependent. Because ζ transforms under Equation (55) nonlinearly (i.e., a shift symmetry), any operators $\mathcal{O}_\Delta$ written in terms of ζ must contain derivatives and are therefore suppressed by powers of aH . This also explains why conservation of ζ fails in multifield inflation, where $\mathcal{O}_\Delta$ can be a nonderivative operator containing a spectator field.

其中例如常数 $c_{1,2}$ 就是依赖模型的。由于 ζ 按照式 (55) 做非线性变换 (即平移对称性)，任何用 ζ 写出的算符 $\mathcal{O}_\Delta$ 都必须包含导数，因此会被 aH 的幂次压低。这也解释了为什么 ζ 的守恒性在多场暴涨中不成立——此时 $\mathcal{O}_\Delta$ 可以是包含旁观场的非导数算符。

Unfortunately, in the UV description, showing that the power counting applies to $\mathcal{O}_\Delta$ in a generic correlation function requires a complicated diagrammatic argument [11]. Without the EFT, we cannot simply count powers of aH to determine the behavior of an operator. With the benefit of hindsight, we know corrections that ruin power counting would have to contribute positive powers of $[aH]$ to cancel the explicit negative powers. Such contributions correspond to power-law divergences in the SdSET and can be removed by an appropriate change of operator basis.

遗憾的是，在紫外描述中，要证明幂计数适用于一般关联函数中的 $\mathcal{O}_\Delta$ ，需要复杂的图论论证 [11]。没有有效场论 (EFT) 的话，我们无法简单通过数 aH 的幂次确定算符的行为。事后回顾我们知道，破坏幂计数的修正必须贡献 $[aH]$ 的正幂次，来抵消显式的负幂次。这类贡献对应慢滚动膨胀有效理论 (SdSET) 中的幂律发散，可以通过适当变换算符基消除。

By comparison, conservation of ζ and γ in SdSET is a relatively trivial observation about the dimension α associated with ζ_+ or γ_+ , defining ζ_+,γ_+ by Equation (24) with $\phi\to\zeta,\gamma$. Concretely, the nonlinearly symmetries must act on the growing modes ζ_+ and γ_+ to be consistent with the long-wavelength limit, $\zeta\to H\gamma_+$ and $\gamma\to H\gamma_+$; yet this symmetry is only possible when $\alpha=0$ for both γ_+ and ζ_+ . In addition, the

EFT cannot contain nonderivative interactions, and therefore ζ and γ are conserved as operators. This also provides the first all-order proof of the conservation of γ_{ij}. Altogether, this implies that in single-field inflation

相比之下，SdSET 中 ζ 和 γ 的守恒性是相对平凡的结论: 关联于 ζ_+ 或 γ_+ 的维度 α，在 $\phi \to \zeta, \gamma$ 的条件下通过式 (24) 定义 ζ_+, γ_+。具体来说，非线性对称性必须作用在增长模式 ζ_+ 和 γ_+ 上才能和长波长极限 $\zeta \to H\gamma_+$ 与 $\gamma \to H\gamma_+$ 自洽；而只有当对 γ_+ 和 ζ_+ 都满足 $\alpha = 0$ 时，这种对称性才存在。此外，该有效场论不能包含非导数相互作用，因此 ζ 和 γ 作为算符是守恒的。这也给出了 γ_{ij} 守恒性的第一个全阶证明。综上，这说明在单场暴涨中

$$\lim_{\mathbf{k}\to 0}\langle\dot{\zeta}(\mathbf{k})\ldots\rangle \to 0 \quad \lim_{\mathbf{k}\to 0}\langle\dot{\gamma}_{ij}(\mathbf{k})\ldots\rangle \to 0, \tag{61}$$

for any correlation function. Note that this is far more restrictive than the behavior of a massless scalar field in de Sitter space. However, the equations are the result of a nonlinearly realized large diffeomorphism and thus will not be apparent for individual terms in the action of ζ or if we introduce a regulator that breaks these symmetries. As discussed in section "Effective Theory in de Sitter," there are few good regulators in de Sitter which is why these all-loop results are not obvious in many direct calculations in the UV description. It would be interesting to revisit the all-order conservation in Melin space where many of these challenges are mitigated [78].

对任意关联函数都成立。注意这比德西特空间中无质量标量场的行为约束性强得多。然而，这些方程是非线性实现的大微分同胚的结果，因此在 ζ 作用量的单独项中，或是当我们引入破坏这些对称性的调节器时，它们并不会显现出来。正如“德西特中的有效理论”一节所讨论的，德西特空间中优良的调节器很少，这就是为什么这些全圈结果在许多紫外描述的直接计算中并不显然。未来在梅林空间重新研究全阶守恒性会是很有意义的方向，那里许多这类挑战都得到了缓解 [78]。

From our discussion of light scalar fields, one is also naturally interested in the time evolution of composite operators. Writing the most general local equation and applying the symmetries in Equations (55) and (56), one finds that the time evolution of ζ^N must be governed by [32]

根据我们对轻标量场的讨论，我们自然也会关注复合算符的时间演化。写出最一般的局部方程并应用式 (55) 和 (56) 中的对称性，可得出 ζ^N 的时间演化必须由下式支配 [32]

$$\frac{\partial}{\partial t}\zeta^N(\mathbf{x},t) = \sum_{n\geq 2}^{N}\gamma_n\left(\begin{array}{c}N\\ n\end{array}\right)\zeta^{N-n}(\mathbf{x},t). \tag{62}$$

Note that for $N = 1$, the right-hand side is zero as needed for conservation of the long modes. Translating this to stochastic inflation gives a generalization of the Fokker-Planck equation,

注意，对于 $N = 1$，长模守恒要求其右侧为零，该式正好满足这一点。将其转换为随机暴涨形式，可得到福克-普朗克方程的推广形式,

$$\frac{\partial}{\partial \mathrm{t}}P(\zeta,\mathrm{t}) = \sum_{n\geq 2}(-1)^n\frac{\gamma_n}{n!}\frac{\partial^n}{\partial\zeta^n}P(\zeta,\mathrm{t}). \tag{63}$$

Note that, in contrast to Equation (44), only derivatives of ζ appear, as we would expect from the shift symmetry. The coefficients γ_n are determined from integrating the connected n-point function

注意，与式 (44) 不同，该式中仅出现 ζ 的导数，这符合平移对称性的预期。系数 γ_n 由连通 n 点函数积分得到

$$\langle \zeta^n (\mathbf{x}=0) \rangle \propto \gamma_n \log [aH] \tag{64}$$

In this concrete sense, the coefficients $\gamma_{n>2}$ characterize the non-Gaussian noise associated with horizon crossing.

从这个明确的意义上来说，系数 $\gamma_{n>2}$ 描述了与视界穿越相关的非高斯噪声。

Unlike an interacting scalar with a potential $V(\phi)$, the solution to the Fokker-Planck equation does not lead to an equilibrium solution. For a free theory, with $\gamma_{n>2}=0$ and $\gamma_2 \equiv \sigma^2 = \Delta_\zeta/\left(2\pi^2\right)$, one finds a solution

与带有势 $V(\phi)$ 的相互作用标量场不同，福克-普朗克方程的解不存在平衡解。对于自由理论，取 $\gamma_{n>2}=0$ 和 $\gamma_2 \equiv \sigma^2 = \Delta_\zeta/\left(2\pi^2\right)$ ，可得到如下解

$$P_{\mathrm{G}}(\zeta, \mathrm{t}, \zeta_0) = \frac{1}{\sqrt{2\pi\sigma^2 \mathrm{t}}} e^{-(\zeta-\zeta_0)^2/(2\sigma^2\mathrm{t})}. \tag{65}$$

Note that this is nothing other than the solution for the evolution of a Gaussian random walk with variance σ and initial conditions $\zeta=\zeta_0$ at $t=0$, as one would expect from the Fokker-Planck equation. In this sense, it is not surprising that there is no equilibrium solution: each mode crosses the horizon and adds a random shift in ζ which adds coherently because $\zeta(\mathbf{k})$ freezes outside the horizon.

注意，这正是方差为 σ 、初始条件为 $\zeta=\zeta_0$ (在 $t=0$ 处) 的高斯随机游走演化的解，符合福克-普朗克方程的预期。从这个意义上说，不存在平衡解并不奇怪: 每个模式穿越视界时都会给 ζ 带来一个随机偏移，由于 $\zeta(\mathbf{k})$ 在视界外冻结，这些偏移会相干叠加。

For an interacting theory, the random walk behavior is generalized to include non-Gaussian noise, as we might have expected from the general Markovian evolution in Equation (45). Introducing $\gamma_{n>2} \neq 0$, we can still find a general solution with initial conditions $\zeta=\zeta_0$ at $t=0$,

对于相互作用理论，随机游走行为会推广到包含非高斯噪声的情况，这和式 (45) 中一般马尔可夫演化的预期一致。引入 $\gamma_{n>2} \neq 0$ 后，我们仍可以找到在 $t=0$ 处满足初始条件 $\zeta=\zeta_0$ 的通解，

$$P(\zeta; \mathrm{t}, \zeta_0) = \exp\left(\sum_{n>2} (-1)^n \frac{\gamma_n \mathrm{t}}{n!} \frac{\partial^n}{\partial \zeta^n}\right) P_{\mathrm{G}}(\zeta; \mathrm{t}, \zeta_0) \tag{66}$$

Notice that around the peak of the distribution, $|\zeta-\zeta_0| \sim \sigma\sqrt{\mathrm{t}}$, we have for $n>2$ and $\mathrm{t} \gg 1$,

注意，在分布峰 $|\zeta-\zeta_0| \sim \sigma\sqrt{\mathrm{t}}$ 附近，对于 $n>2$ 和 $\mathrm{t} \gg 1$ ，我们有

$$\frac{\gamma_n t}{n!} \frac{\partial^n}{\partial \zeta^n} P_G(\zeta; t, \zeta_0) = \mathcal{O}\left(\gamma_n t^{1-n/2} \sigma^{-n}\right) P_G(\zeta; t, \zeta_0) \ll P_G(\zeta; t, \zeta_0) \tag{67}$$

We see that after a large number of efolds (t ⍰ 1), the distribution tends to a Gaussian, as we would expect from the central limit theorem. As a result, the behavior of the probability distribution of ζ using stochastic inflation is again improved compared to standard perturbation theory. In this case, rather than a nontrivial fixed point in the form of an equilibrium distribution, the fluctuations of ζ approach the behavior of a Gaussian random walk (We can understand the difference between the two cases from the fact that the potential is a relevant deformation of a random walk.). Importantly, even though the probability distribution is a Gaussian, it is distinct from the probability distribution of the in-in theory due to the additional by powers of t.

我们可以看到，经过大量 e 折叠数 (t ⍰ 1) 后，分布趋近于高斯分布，这符合中心极限定理的预期。因此，利用随机暴涨得到的 ζ 概率分布行为，相比标准微扰论确实得到了改进。在这种情况下，ζ 的涨落没有呈现为平衡分布形式的非平凡不动点，而是趋近于高斯随机游走的行为 (两种情况的差异可以理解为: 势是随机游走的相关形变)。重要的是，即使该概率分布是高斯分布，它也和入理论 (in-in theory) 的概率分布不同，因为它额外带有 t 的幂次项。

Implicit in this discussion is that we can define the time, t, at which the fluctuations are measured. For inflation itself, when the background classical evolution of the scalar field defines the end of inflation, we can define observables on the ϕ = constant surface, which characterizes the end of inflation or reheating. However, when the quantum fluctuations of ϕ , or alternatively ζ , become large enough to overwhelm the classical evolution, the lack of non-perturbative observables in de

该讨论隐含了一个前提: 我们可以定义测量涨落的时间 t。对于暴涨本身，当标量场的背景经典演化确定了暴涨的终点后，我们可以在 $\phi =$ 常数曲面上定义可观测量，该曲面表征了暴涨结束或再加热的时刻。然而，当 ϕ (或 ζ) 的量子涨落大到足以压倒经典演化时，德西特空间中就缺少非微扰可观测量

Sitter space ultimately limits our understanding. It was argued in [36] that there is a phase transition that occurs between these two regimes, where the volume of the reheating surface, V , is the order parameter. When $\langle V \rangle < \infty$, given a finite initial volume, then the reheating surface is well defined, and statistics on that surface are well-defined in the semiclassical sense. As we change the parameters of inflation such that $\langle V \rangle \to \infty$, eternal inflation occurs, and the reheating surface is no longer well-defined. The benefit of this approach is that one can define the onset of eternal inflation from the regime that is under control, without requiring that we define observables directly in the eternally inflating regime.

德西特空间最终会限制我们的认知。文献 [36] 指出，这两个区域之间会发生相变，其中再加热曲面的体积 V 是序参量。当 $\langle V \rangle < \infty$ 时，给定初始有限体积，再加热曲面是明确定义的，且该曲面上的统计量在半经典意义上也是良定义的。当我们改变暴胀参数使得 $\langle V \rangle \to \infty$ 时，永恒暴胀发生，再加热曲面不再是良定义的。该方法的优势在于，我们可以从可控区域定义永恒暴胀的开端，无需直接在永恒暴胀区域定义可观测量。

The phase transition to eternal inflation can be defined using the above probability distribution. In the limit of vanishing slow-roll parameters, the relationship between the evolution of the scalar field driving inflation and the metric fluctuation becomes $\phi = \dot{\phi}_0 (\mathrm{t} - \zeta)/H$ where $\dot{\phi}_0$ is a constant. The probability that reheating occurs at time t, $p_{\mathrm{R}}(t)$, is determined the number of points that reach the end point of inflation, $\phi = \phi_c$, at time t that had previously been inflating, $\phi < \phi_c$, for all $\mathrm{t}' < \mathrm{t}$. To simplify the discussion, we can set $\phi_c = 0$, and therefore the end of inflation corresponds to $\zeta = \mathrm{t}$. Assuming we can neglect all the non-Gaussian in terms in Equation (63), the probability distribution for ϕ or ζ is given by Equation (65) but where we impose absorbing boundary conditions at $\zeta = t$, as inflation ends when ζ reaches this point,

通往永恒暴胀的相变可以用上述概率分布定义。在慢滚参数趋近于零的极限下，驱动暴胀的标量场演化与度规涨落之间的关系为 $\phi = \dot{\phi}_0 (\mathrm{t} - \zeta) / H$ ，其中 $\dot{\phi}_0$ 是常数。在时刻 t, $p_{\mathrm{R}}(t)$ 发生再加热的概率，由所有满足条件的点的数量决定: 这些点在时刻 t 到达暴胀终点 $\phi = \phi_c$ ，且此前一直在 $\mathrm{t}' < \mathrm{t}$ 范围内维持暴胀 $\phi < \phi_c$ 。为简化讨论，我们可以令 $\phi_c = 0$ ，因此暴胀的终点对应 $\zeta = \mathrm{t}$ 。假设我们可以忽略式 (63) 中所有非高斯项，那么 ϕ 或 ζ 的概率分布由式 (65) 给出，但我们需要在 $\zeta = t$ 处施加吸收边界条件，因为当 ζ 到达该点时暴胀就结束了，

$$P_{\mathrm{G}} (\phi [\zeta] < 0, \mathrm{t}, \zeta_0) = \frac{1}{\sqrt{2\pi\sigma^2 \mathrm{t}}} \left[e^{-(\zeta - \zeta_i)^2 / (2\sigma^2 \mathrm{t})} - e^{-4\zeta_i / (2\sigma^2)} e^{-(\zeta + \zeta_0)^2 / (2\sigma^2 \mathrm{t})} \right],$$

(68)

where again ζ_0 is the initial value of ζ at $\mathrm{t} = 0$. Following [36], the reheating probability is

其中 ζ_0 仍是 ζ 在 $\mathrm{t} = 0$ 处的初始值。根据文献 [36]，再加热概率为

$$p_{\mathrm{R}} (\mathrm{t}) = -\frac{\mathrm{d}}{\mathrm{dt}} \int_{-\infty}^{0} \mathrm{d}\phi P_{\mathrm{G}} (\phi; \mathrm{t}) = -\frac{\mathrm{d}}{\mathrm{dt}} \int_{-\infty}^{\phi_c} \mathrm{d}\zeta P (\zeta; \mathrm{t}) \propto e^{-\mathrm{t}/(2\sigma^2)}, \tag{69}$$

where we used the Fokker-Planck equation and integrated by parts. The order parameter for eternal inflation is the average reheating volume,

推导中我们使用了福克尔-普朗克方程并做了分部积分。永恒暴胀的序参量是平均再加热体积,

$$\langle V \rangle_{\mathrm{G}} = L^3 \int_0^{\infty} \mathrm{dt} e^{3\mathrm{t}} p_{\mathrm{R,G}} (t) \simeq L^3 \int_0^{\infty} \mathrm{dt} e^{\mathrm{t}(3 - 1/(2\sigma^2))}, \tag{70}$$

where L^3 is the size of the initial patch at $\mathrm{t} = 0$. One then defines the phase transition to eternal inflation as the value of σ (or $H^2/\dot{\phi}_0$) where the average volume of the reheating surface diverges or

其中 L^3 是 $\mathrm{t} = 0$ 处初始 patch 的尺寸。我们将通往永恒暴胀的相变定义为平均再加热曲面体积发散对应的 σ (或 $H^2/\dot{\phi}_0$) 取值，即

$$\sigma^2 = \frac{\Delta_\zeta}{2\pi^2} > \frac{1}{6}. \tag{71}$$

Although this does not give us any direct insight into the correct physical description of the eternal inflating regime, it gives us a sharp notion of where the boundary lies which controls our observables when the reheating volume is finite.

虽然这无法给我们提供关于永恒暴胀区域正确物理描述的任何直接见解，但它能明确给出边界位置，当再加热体积有限时，我们的可观测量由该边界控制。

Naturally, one might wish to revisit this calculation in the presence of nontrivial interactions during inflation. Naively, one might imagine the result is largely unchanged, again due to the central limit theorem. However, the eternal inflation regime is defined by $|\zeta - \zeta_0| \propto t$, which is not a typical fluctuation. If we repeat our power counting in this regime, we have for $n > 2$ and $\mathrm{t} \gg 1$

自然，人们可能希望重新计算暴胀过程存在非平凡相互作用时的结果。从直觉上看，由于中心极限定理，结果大概率不会有太大变化。但永恒暴胀区域是由 $|\zeta-\zeta_0| \propto t$ 定义的，这并非典型涨落。如果我们在该区域重新做幂次计数，可得对于 $n>2$ 和 $\mathrm{t} \gg 1$ 有

$$\frac{\gamma_n \mathrm{t}}{n!}\frac{\partial^n}{\partial \zeta^n} P_{\mathrm{G}}(\zeta;\mathrm{t},\zeta_0) = \mathcal{O}\left(\gamma_n \mathrm{t}\sigma^{-2n}\right) P_{\mathrm{G}}(\zeta;\mathrm{t},\zeta_0). \tag{72}$$

This contribution can easily dominate over the Gaussian term, even a model with small fluctuations $\sigma \ll 1$, and is weakly coupling at horizon cross $(\gamma_n\sigma^{-n} \ll 1)$. These models are known to have weakly coupled UV completions [14] which suggests the breakdown is not due to strong coupling in the model of inflation but is instead a breakdown of the EFT description of inflation or stochastic inflation when calculating the tail of the probability distribution. This phenomena is not unique to cosmology but is a reflection of the large deviation principle [95], which characterizes random walks that are linear in time (or the number of steps). These large fluctuations are sensitive to the microscopic details of the walk and do not follow the central limit theorem. Improving our understanding of rare fluctuations and the origin of the breakdown of EFT is important for both theoretical questions about cosmology, like the nature of eternal inflation and the meaning of the de Sitter entropy [40], and observational consequences, like the production rate of primordial black holes [98].

这一贡献可以轻松超过高斯项，哪怕是在小涨落模型 $\sigma \ll 1$ 中，它在视界穿越时也仍是弱耦合 $(\gamma_n\sigma^{-n} \ll 1)$ 。已知这些模型存在弱耦合的紫外完备性 [14]，这说明该破缺并非来自暴胀模型中的强耦合，而是在计算概率分布尾部时，暴胀或随机暴胀的有效场论描述本身发生了破缺。该现象并非宇宙学独有，而是大偏差原理 [95] 的体现，大偏差原理描述了对时间 (或步数) 线性的随机游走。这类大涨落对游走的微观细节敏感，不服从中心极限定理。增进我们对稀有涨落和有效场论破缺起源的理解，无论是对宇宙学理论问题 (如永恒暴胀的本质和德西特熵的意义 [40])，还是对观测效应 (如原初黑洞的产生率 [98]) 都十分重要。

Conclusions

结论

The physics of de Sitter space presents a significant challenge to our understanding of quantum gravity [9, 94, 108] . There is no boundary where metric fluctuations decouple in which we can define observables, raising the question of whether there are any non-perturbative observables in a quantum theory of de Sitter space [5,21]. It has been suggested that the more basic technical challenges associated with de Sitter, like IR divergences and secular growth, are tied to these serious non-perturbative questions, even though they are not necessarily related.

德西特空间物理学对我们关于量子引力的认知提出了重大挑战 [9, 94, 108] 。不存在度量涨落退耦、可用来定义可观测量的边界，这引出了一个问题: 德西特空间量子理论中是否真的存在非微扰可观测量 [5,21]。已有观点指出，和德西特相关的更基础技术性问题，比如红外发散和长期增长，都和这些严肃的非微扰问题绑定，即便它们未必存在关联。

Effective field theory gives us a more precise language to discuss the successes and failures of QFT and quantum gravity in de Sitter or inflationary backgrounds. Quantum field theory and perturbative quantum gravity on a fixed de Sitter background are characterized by a single energy scale, H , the rate of expansion in the cosmological slicing. The influence of interactions on the statistics of the long-wavelength fluctuations is determined by flat space power counting, where H plays the role of the center of mass enter. This can also be understood from the perspective of the static patch, where the local physics looks thermal with an effective temperature $T_{\mathrm{dS}} = H/(2\pi)$ [48]. From the EFT point of view, the litany of problems often associated with de Sitter itself is often just the result of lacking good regulators for QFT in cosmological backgrounds. Without such a regulator, short-distance power-law divergences introduce order-one shifts to the couplings and require careful treatment. Recent progress in calculating cosmological correlators using Mellin space [78, 86, 87] or from analytic continuation from AdS [68, 88] might circumvent these challenges and make the implementation of basic EFT in dS straightforward.

有效场论为我们提供了更精确的语言，来讨论量子场论与量子引力在德西特背景或暴胀背景下的成败。固定德西特背景下的量子场论与微扰量子引力由单一能尺度量，H 即宇宙切片中的膨胀速率。相互作用对长波涨落统计的影响由平坦空间幂次计数决定，其中 H 扮演质心系能量的角色。这一点也可以从静态补丁的视角理解: 静态补丁中的局域物理呈现温度为 $T_{\mathrm{dS}} = H/(2\pi)$ 的热性质 [48]。从有效场论的角度看，通常和德西特本身绑定的诸多问题，往往只是因为宇宙学背景下的量子场论缺乏合适的正则化工具。没有这类正则化工具的话，短距离幂律发散会给耦合带来一阶偏移，需要谨慎处理。近期利用梅林空间 [78, 86, 87] 或从反德西特空间解析延拓 [68, 88] 计算宇宙关联函数的进展，或许能绕过这些挑战，让基本有效场论在德西特空间中的应用变得直接。

While better regulators can resolve challenges associated with short distances, it does not fully explain all the divergences of loop corrections at late times and large distances. In the language of EFT, any such large effect should be understandable in terms of power counting, where interactions that are important (negligible) on super-horizon scales are relevant (irrelevant) in the renormalization group sense. This is not possible in the conventional QFT description and requires rewriting the theory in terms of scaling operators of the superhorizon description. The soft de Sitter effective theory provides such a description where we can understand phenomena ranging from stochastic inflation to conservation of the adiabatic modes in terms of power counting and symmetries.

尽管更好的正则化工具可以解决和短距离相关的挑战，但它无法完全解释晚时间、大距离处所有圈修正的发散。用有效场论的语言来说，任何这类大效应都应当可以通过幂次计数理解: 超视界尺度上重要 (可忽略) 的相互作用，在重整化群意义上就是 relevant(无关) 算符。这在传统量子场论描述中无法实现，需要用超视界描述的标度算符重新改写理论。软德西特有效理论就提供了这样一种描述，让我们可以通过幂次计数和对称性理解从随机暴胀到绝热模守恒的一系列现象。

The ultimate questions about de Sitter and inflation are expected to take us far from a fixed background with small fluctuations. Eternal inflation [53, 62, 93, 100] , for example, is a poorly characterized cosmological phase where it is difficult to even define observables [45]. Part of the power of EFT is that it predicts its own demise, usually in terms of simple power counting. By reshaping our understanding of de Sitter to make power counting manifest, we hope to make the challenges that remain more transparent and familiar. In addition, by isolating the essential long-distance degrees of freedom, these problems may appear more trackable and open the door to new approaches and solutions.

关于德西特与暴胀的终极问题，其研究范围本就远超出带微小涨落的固定背景。例如永恒暴胀 [53, 62, 93, 100] 就是一个刻画不足的宇宙学相，在该相中我们甚至很难定义可观测量 [45]。有效场论的一大特点就是，它通常能通过简单的幂次计数预言自身的适用边界。通过重塑我们对德西特的理解让幂次计数变得清晰，我们希望能让余下的挑战更明晰、更易于处理。此外，通过分离核心长距离自由度，这些问题或许会变得更易于追踪，并为新方法和新解决方案敞开大门。

Cross-References

交叉引用

Effective Field Theory for Large-Scale Structure

大尺度结构有效场论

Gravity, Horizons, and Open EFTs

引力、视界与开放有效场论

Quantum General Relativity and Effective Field Theory

量子广义相对论与有效场论

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